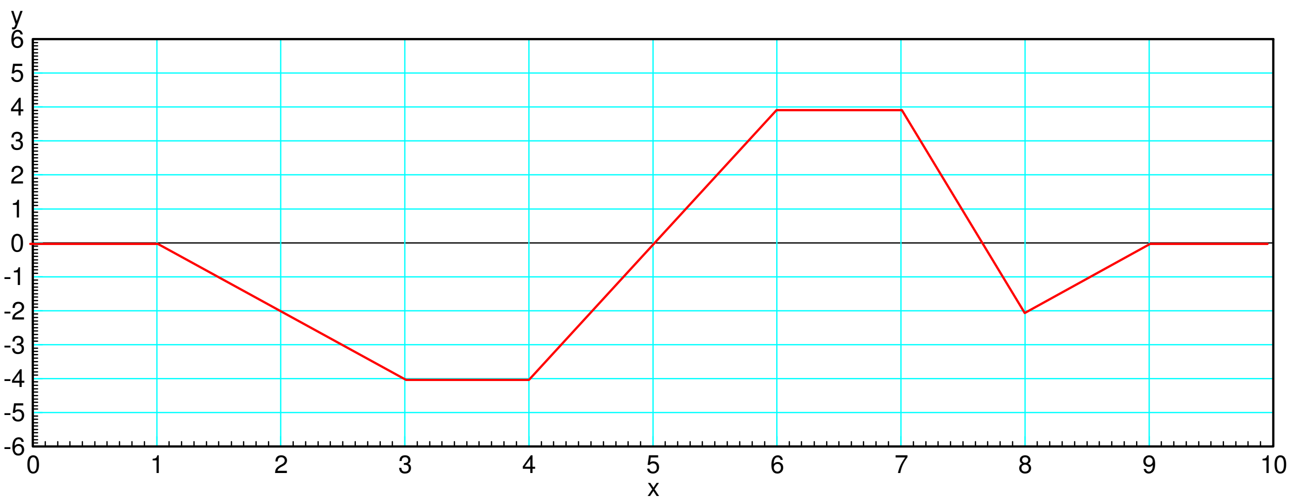
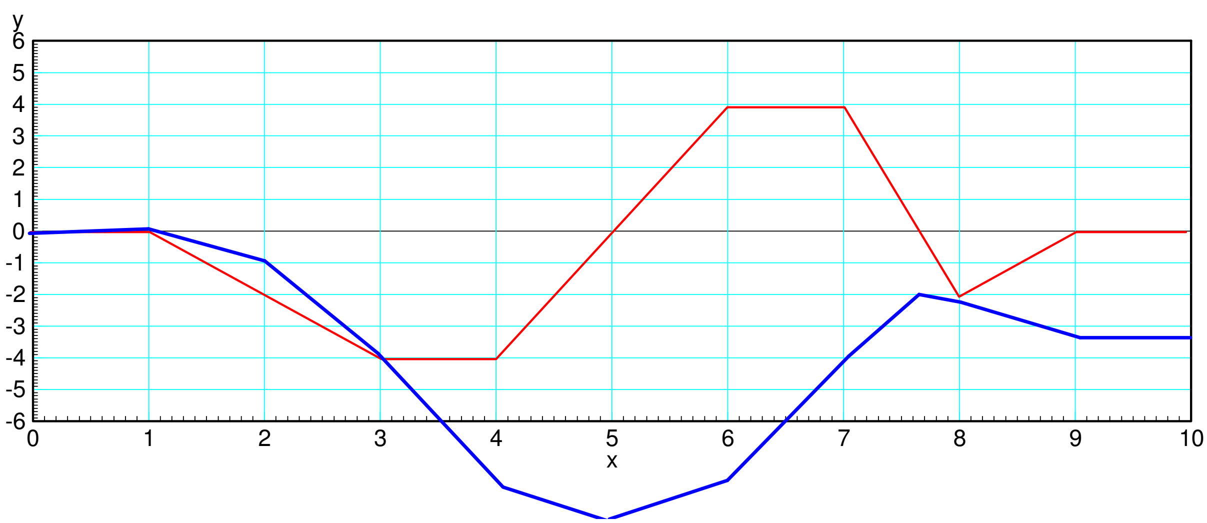
# ECE 111 - Homework #7:

Math 166: Integration
Due Tuesday, October 10th
Please submit via email, via hard copy, or on BlackBoard

1) Sketch the integral of the following funciton




*If this is how much money you are depositing (positive) or withdrawing (negative) from your checking account, what is the balance at each instance?*



## Numerical Integration

2) Use numerical methods to determine the integral of y

$$y = \left(\frac{\cos(x)}{x^2+0.5}\right)$$

$$z = \int y \cdot dx$$

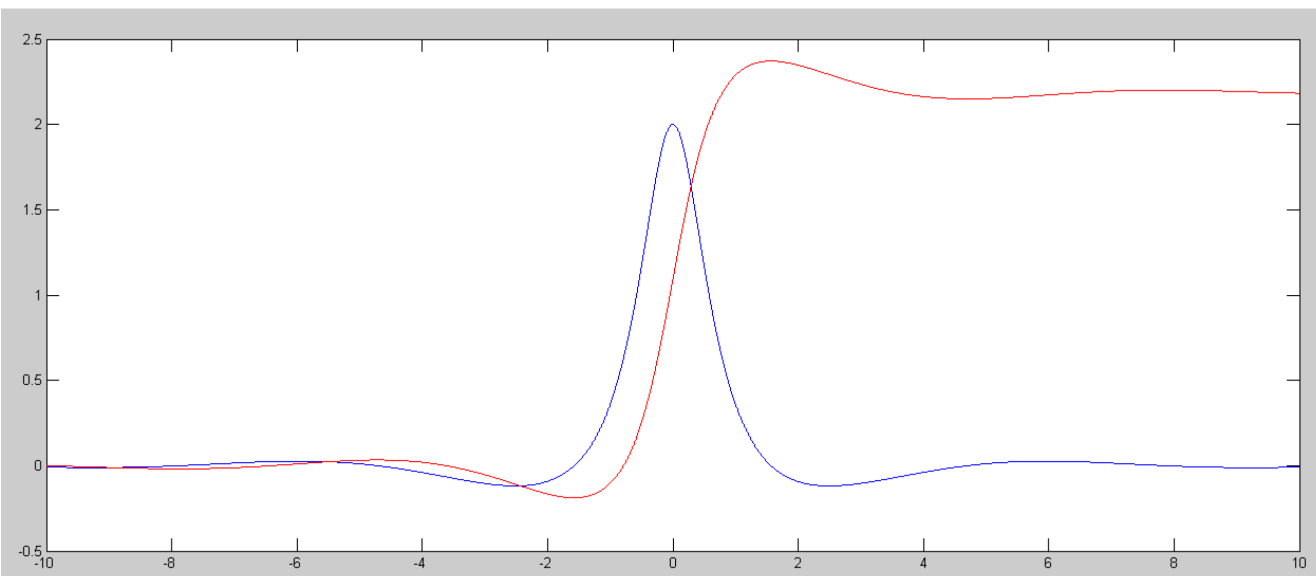
for -10 < x < 10. ( a plot is sufficient ). Assume z(-10) = 0.

Start with a function for integration:

```
function [y ] = Integrate( x, dy )

   npt = length(x);
   y = 0*dy;
   for i=2:npt
      y(i) = y(i-1) + 0.5*(dy(i) + dy(i-1)) * (x(i) - x(i-1));
      end
   end
```

Call it from Matlab

```
>> x = [-10:0.001:10]';
>> y = cos(x) ./ (x.^2 + 0.5);
>> z = Integrate(x,y);
>> plot(x,y,'b',x,z,'r')
>> xlabel('x')
>>
```




y(x) (blue) and the integral of y(x) (red)

Comment: With Matlab and numerical techniques, it doesn't matter if you can or can't find the integral by hand. If you can get the funciton into Matlab, it can find the integral using numerical techniques.

3) Use numerical methods to determine the integral of y

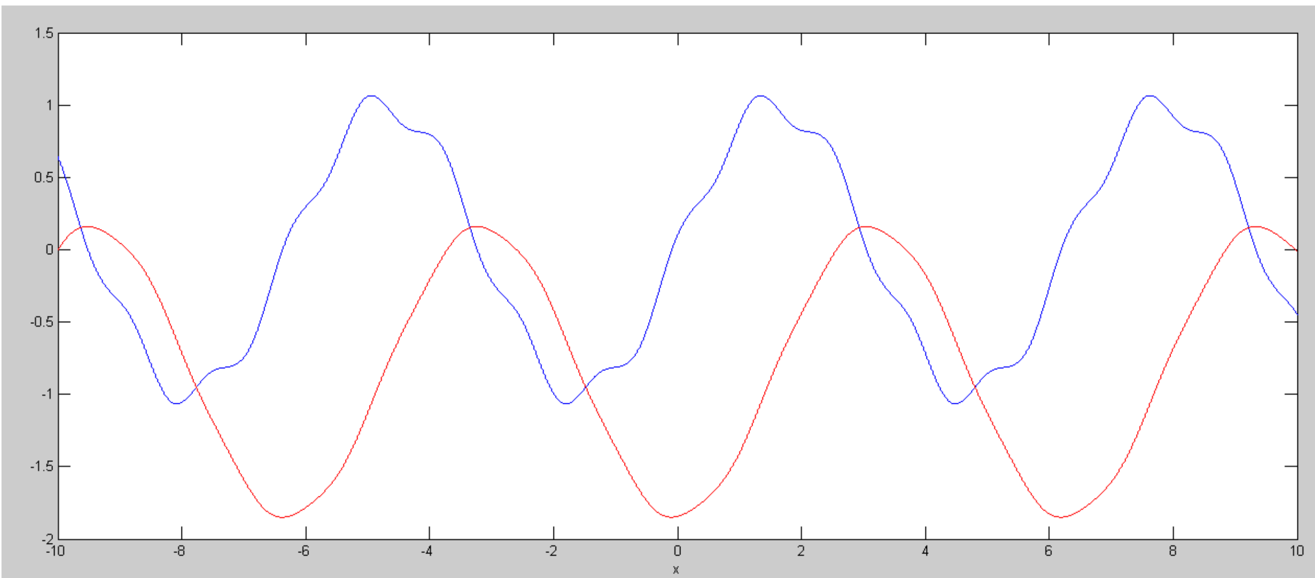
$$y = \sin(x) + 0.1\cos(5x)$$

$$z = \int y \cdot dx$$

for -10 < x < 10. ( a plot is sufficient ). Assume z(-10) = 0.

Same procedure as before. From the command window:

```
>> x = [-10:0.001:10]';
>> y = sin(x) + 0.1*cos(5*x);
>> z = Integrate(x,y);
>> plot(x,y,'b',x,z,'r')
>> xlabel('x')
>>
```




Comment:

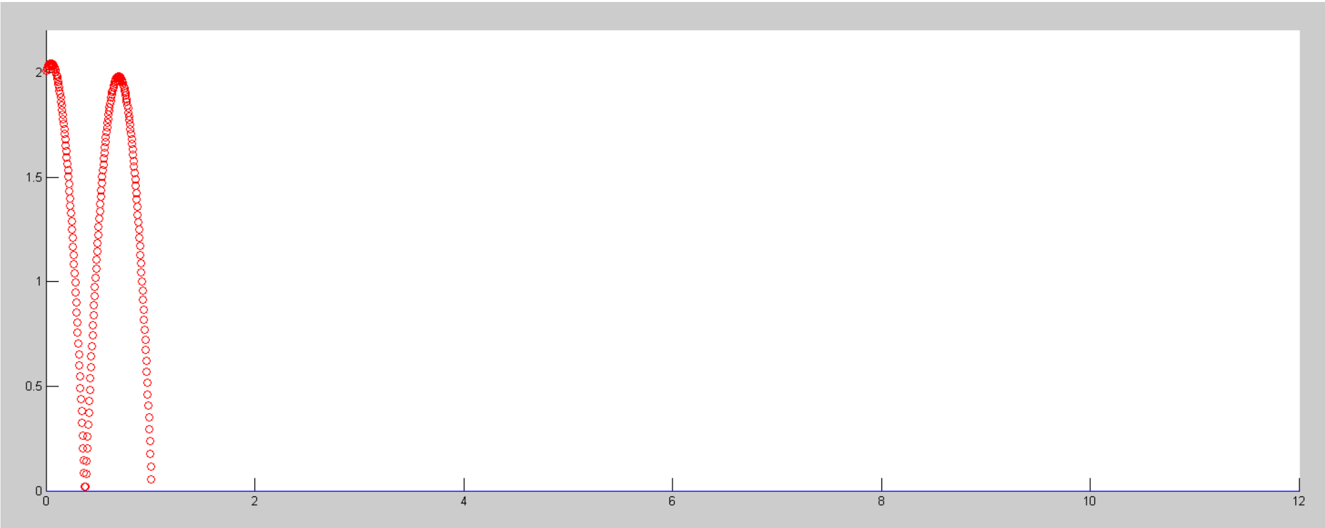
- Integration attenuates high-frequency terms
- Integration helps remove noise from a signal

## Animation in Matlab with Numerical Integration

4) Calculate the (x,y) position of a bouncing ball in freefall:

- The acceleration is y'' = -9.8 m/s2
- If the ball hits the ground (y<0) the velocity becomes positive: y' = |y'|
- The initial position is (x = 0, y = 2)
- The initial velocity is (x' = 0.5, y' = 0)

Plot the path of the ball up to its second bounce




Matlab Code

```
x = 0;
y = 2;

dx = 0.5;
dy = 1;

t = 0;
dt = 0.01;
hold on

Bounce = 0;

while(Bounce < 2)
    ddx = 0;
    ddy = -9.8;

    dx = dx + ddx * dt;
    dy = dy + ddy * dt;

    x = x + dx * dt;
    y = y + dy * dt;

    t = t + dt;

    if(y < 0)
        dy = abs(dy);
        Bounce = Bounce + 1;
        end
```

```
    plot(x,y,'ro',[0,12],[0,0],'b',0,10,'b+');
    xlim([0,12]);
    ylim([0,2.2]);

    pause(0.01);
end

x
```

5) Determine the initial velocity on x' so that the ball hits a target at (x=10, y=0) on the second bounce

- note: this is a f(x) = 0 problem

Use California method

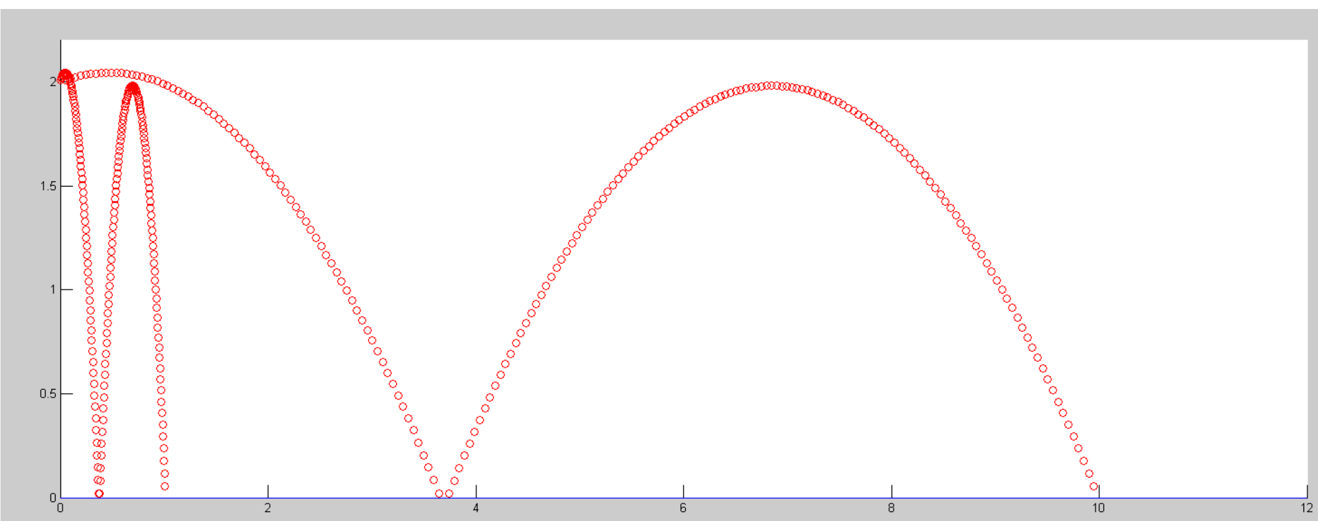
x' = 0.5  results in x = 1.015 at the 2nd bounce

For x(bounce #2) = 10

$$x' = \left(\frac{10m}{1.015m}\right)0.5\frac{m}{s}$$

$$x' = 4.926\frac{m}{s}$$

This results in the 2nd bounce being 9.9998 meters

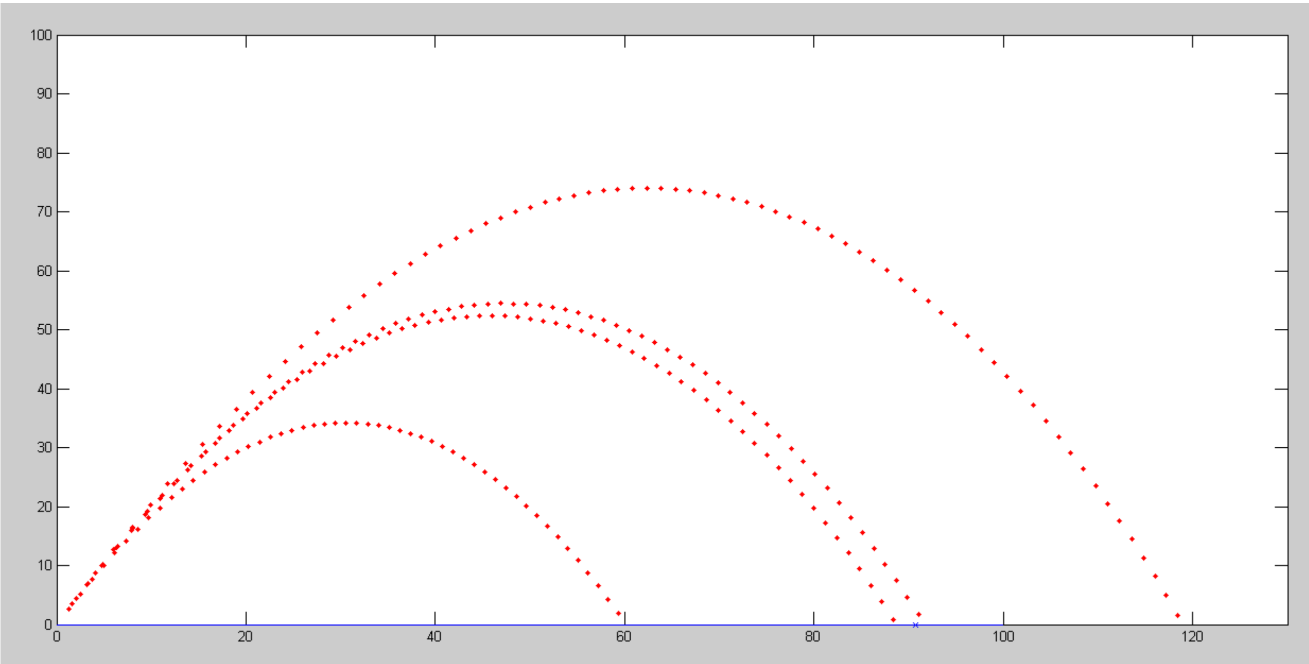


**f(x) = 0: Shoot Game**

- Pick a random number from 50 to 100 for your target.
- Pick a random number from 30 to 70 for your firing angle

6) Use trial and error to find the initial velocity (X) to fire a tennis ball to hit the target (result is zero)

```
>> Target = 50 * rand + 50

Target =    90.7362

>> Angle = 50*rand + 20

Angle =   65.2896

>> Shoot(30, Angle, Target)

ans =   30.6515

>> Shoot(50, Angle, Target)

ans =  -28.2262

>> Shoot(40, Angle, Target)

ans =   -0.9533

>> Shoot(39, Angle, Target)

ans =    2.0654

>>
```



7) Repeat using Newton's method to find the initial velocity (X) to fire the tenis ball to hit the target

Using California method: (There are better methods than guessing)

```
>> X1 = 0;
>> Y1 = Target;
>> X2 = 30;
>> Y2 = Shoot(X2, Angle, Target)

Y2 =   30.6515

>> X3 = X2 - (X2-X1)/(Y2-Y1)*Y2

X3 =   45.3041

>> Y3 = Shoot(X3, Angle, Target)

Y3 =  -16.1212

>> X4 = X3 - (X3-X2)/(Y3-Y2)*Y3

X4 =   40.0292

>> Y4 = Shoot(X4, Angle, Target)

Y4 =   -1.0409

>> X5 = X4 - (X4-X3)/(Y4-Y3)*Y4

X5 =   39.6651

>> Y5 = Shoot(X5, Angle, Target)

Y5 =    0.0524
```