ECE 111 - Homework #2

Math 103 - Algebra, Functions & Solving f(x) = 0. Due Tuesday, September 5th
Please submit via email, via hard copy, or on BlackBoard

Newton's Method

1) Let x and y be related by:

$$y = x^3 - x^2 - 6x + 1$$

Use Newton's method to solve for x when

- y = 0
- y = 10

First, create a Matlab function to compute y (not 100% necessary)

```
function [y] = fn(x)
   y = x^3 - x^2 - 6*x + 1;
end
```

Write a Matlab script to itterate to find the solution for y=0.

- Your initial guess for x usually dictates which solution you converge to

```
y0 = 0;
x3 = 10.1;

for n=1:10
   x1 = x3;
   y1 = fn(x1) - y0;
   x2 = x3 + 0.01;
   y2 = fn(x2) - y0;

   x3 = x1 - (x2-x1)/(y2-y1)*y1;
   disp([n x1, fn(x1)]);
end
```

Result: x = 2.9308 (there are other solutions)

n	x	y(x)
1.0000	10.1000	868.6910
2.0000	6.9989	252.8598
3.0000	5.0103	71.6101
4.0000	3.8054	18.7915
5.0000	3.1776	3.9225
6.0000	2.9600	0.4125
7.0000	2.9314	0.0087
8.0000	2.9308	0.0001
9.0000	2.9308	0.0000
10.0000	2.9308	0.0000

Write a Matlab script to itterate to find the solution for y = 10:

```
y0 = 10;
x3 = 10.1;

for n=1:10
   x1 = x3;
   y1 = fn(x1) - y0;
   x2 = x3 + 0.01;
   y2 = fn(x2) - y0;

   x3 = x1 - (x2-x1)/(y2-y1)*y1;
   disp([n x3, fn(x3)]);
end
```

Result: x = 3.4734

n	x	y(x)
1.0000	10.1000	868.6910
2.0000	7.0346	257.4174
3.0000	5.1105	77.6913
4.0000	4.0235	25.8050
5.0000	3.5671	12.2615
6.0000	3.4771	10.0865
7.0000	3.4734	10.0005
8.0000	3.4734	10.0000
9.0000	3.4734	10.0000
10.0000	3.4734	10.0000

2) Let x and y be related by

$$y = \sin(2x)$$

$$y = (x+1)(x-1)$$

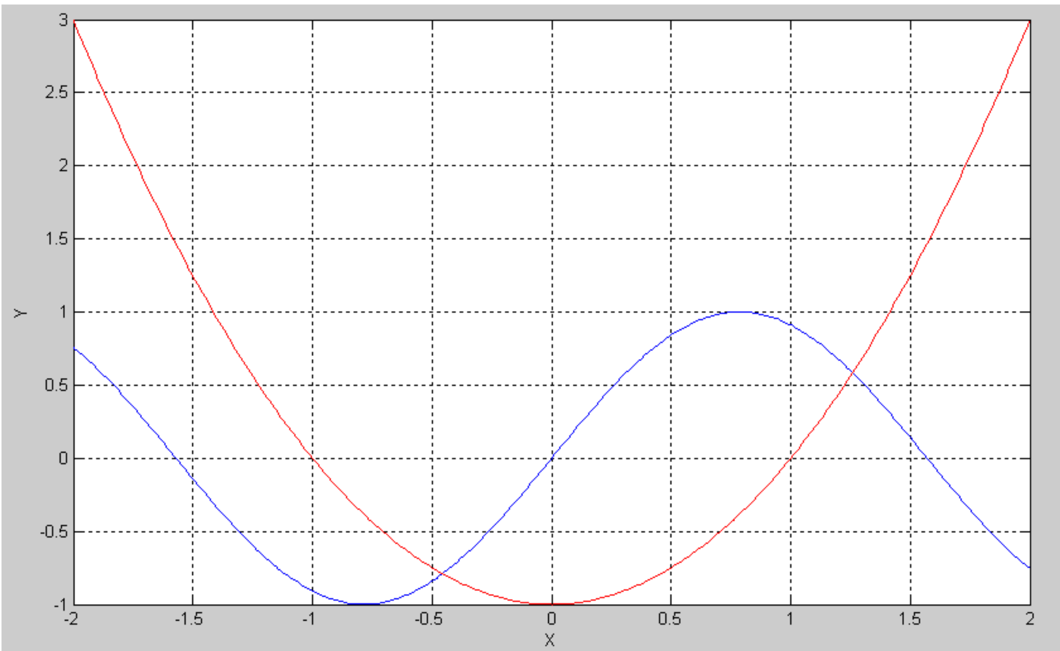
Find all solutions using graphical methods. (Plot both functions on the same graph. The solution is when the two functions intersect.)

In the Matlab command window, plot the two functions on the same graph

```
>> x = [-2:0.01:2]';
>> y1 = sin(2*x);
>> y2 = (x+1) .* (x-1);
>> plot(x,y1,'b',x,y2,'r');
>> grid
>> xlabel('X');
>> ylabel('Y');
>>
```

From the graph, there are two solutions:

- x = -0.4, y = .03
- x = +1.25, y = +0.6



3) Find the solutions to problem #2 using Newton's method.

Let

$$y_1 = \sin(2x)$$

$$y_2 = (x+1)(x-1)$$

$$e = y_1 - y_2$$

Find the solutions for f(x) = 0 using Netwon's method.

First, modify the function from problem #1

```
function [y] = fn(x)
   y1 = sin(2*x);
   y2 = (x+1)*(x-1);
   y = y1 - y2;
end
```

Use Newton's method to find the answer. Starting at x = -0.4:

```
x3 = -0.4;

for n=1:10
   x1 = x3;
   y1 = fn(x1);
   x2 = x3 + 0.01;
   y2 = fn(x2);

   x3 = x1 - (x2-x1)/(y2-y1)*y1;
   disp([n x1, fn(x1)]);
end
```

n	x	y1 - y2
1.0000	-0.4000	0.1226
2.0000	-0.4558	0.0017
3.0000	-0.4566	0.0000
4.0000	-0.4566	0.0000
5.0000	-0.4566	0.0000
6.0000	-0.4566	0.0000
7.0000	-0.4566	0.0000
8.0000	-0.4566	0.0000
9.0000	-0.4566	0
10.0000	-0.4566	0

Starting at x = +1:

```
x3 = 1.0;

for n=1:10
   x1 = x3;
   y1 = fn(x1);
   x2 = x3 + 0.01;
   y2 = fn(x2);

   x3 = x1 - (x2-x1)/(y2-y1)*y1;
   disp([n x1, fn(x1)]);
end
```

n	x	y1-y2
1.0000	1.0000	0.9093
2.0000	1.3179	-0.2523
3.0000	1.2606	-0.0078
4.0000	1.2587	-0.0000
5.0000	1.2587	-0.0000
6.0000	1.2587	-0.0000
7.0000	1.2587	-0.0000
8.0000	1.2587	-0.0000
9.0000	1.2587	0.0000
10.0000	1.2587	0.0000

The two solutions are:

- x = -0.4566
- x = 1.2587

Newton's Method with a Thermistor

Assume the temperature - resistance relationship of a thermistor is:

$$R = 1000 \cdot \exp\left(\frac{3905}{T+273} - \frac{3905}{298}\right)\Omega$$

$$e = R - R_0$$

```
T = [-20:0.5:30]';
R = 1000*exp( 3905./(T+273) - 3905/298 );
plot(T,R);
```

4) Write a Matlab funciton which

- Is passes the temeprature T, and
- Returns e (the difference between R and R0)

Changing the problem: Return the resistanec at T

```
function [R] = Therm(T)
   R = 1000 * exp(3905 / (T+273) - 3905/298);
   end
```

5) Use Newton's method to find the temperature when

- R0 = 2000 Ohms
- R0 = 5000 Ohms

Set up a Matlab script

```
R0 = 2000;
x3 = 0;

for n=1:10
   x1 = x3;
   y1 = Therm(x1) - R0;
   x2 = x3 + 0.01;
   y2 = Therm(x2) - R0;

   x3 = x1 - (x2-x1)/(y2-y1)*y1;
   disp([n x1, Therm(x1)]);
end
```

R0 = 2000:

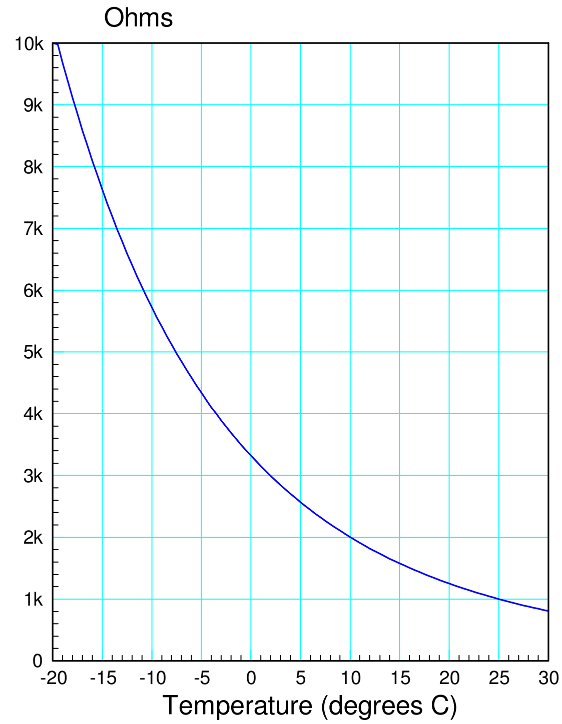
- T = 10.029C

n	T	R
1.0000e+000	0	3.3201e+003
2.0000e+000	7.5909e+000	2.2547e+003
3.0000e+000	9.8694e+000	2.0156e+003
4.0000e+000	1.0028e+001	2.0001e+003
5.0000e+000	1.0029e+001	2.0000e+003
6.0000e+000	1.0029e+001	2.0000e+003
7.0000e+000	1.0029e+001	2.0000e+003
8.0000e+000	1.0029e+001	2.0000e+003
9.0000e+000	1.0029e+001	2.0000e+003
1.0000e+001	1.0029e+001	2.0000e+003

R0 = 5000

- T = -7.5968C

n	T	R
1.0000e+000	0	3.3201e+003
2.0000e+000	-9.6595e+000	5.6108e+003
3.0000e+000	-7.7258e+000	5.0359e+003
4.0000e+000	-7.5973e+000	5.0001e+003
5.0000e+000	-7.5968e+000	5.0000e+003
6.0000e+000	-7.5968e+000	5.0000e+003
7.0000e+000	-7.5968e+000	5.0000e+003
8.0000e+000	-7.5968e+000	5.0000e+003
9.0000e+000	-7.5968e+000	5.0000e+003
1.0000e+001	-7.5968e+000	5.0000e+003



Newton's Method and a Voltage Divider

Assume

$$R = 1000 \cdot \exp\left(\frac{3905}{T+273} - \frac{3905}{298}\right)\Omega$$

$$V = \left(\frac{R}{R+1000}\right) \cdot 10V$$

$$e = V - V_0$$

6) Write a Matlab function which

- Is passed the temperature, T, and
- Returns the error, e.

Modify the previous program to return V:

```
function [V] = Therm(T)
   R = 1000 * exp(3905 / (T+273) - 3905/298);
   V = R / (R + 1000) * 10;
end
```

7) Use Netwon's method to determine the temperature when

- V0 = 8.00V
- V0 = 6.00V

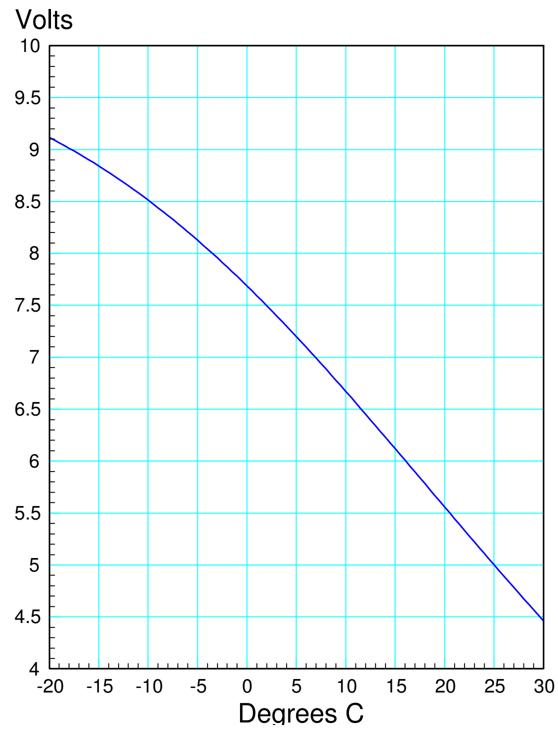
Modify the previous program to operate on voltage

```
V0 = 8.00;
x3 = 0;

for n=1:10
   x1 = x3;
   y1 = Therm(x1) - V0;
   x2 = x3 + 0.01;
   y2 = Therm(x2) - V0;

   x3 = x1 - (x2-x1)/(y2-y1)*y1;
   disp([n x1, Therm(x1)]);
end
```

For V = 8.00V

- T = -3.5098C

n	T	V
1.0000	0	7.6853
2.0000	-3.3764	7.9885
3.0000	-3.5095	8.0000
4.0000	-3.5098	8.0000
5.0000	-3.5098	8.0000
6.0000	-3.5098	8.0000
7.0000	-3.5098	8.0000
8.0000	-3.5098	8.0000
9.0000	-3.5098	8.0000
10.0000	-3.5098	8.0000



For V = 6.00V

- T = 16.0560C

n	T	V
1.0000	0	7.6853
2.0000	18.0785	5.7728
3.0000	16.0580	5.9998
4.0000	16.0560	6.0000
5.0000	16.0560	6.0000
6.0000	16.0560	6.0000
7.0000	16.0560	6.0000
8.0000	16.0560	6.0000
9.0000	16.0560	6.0000
10.0000	16.0560	6.0000