ECE 111 - Homework #15

ECE 343 Signals- Due Tuesday, December 5th
Please email to jacob.glower@ndsu.edu, or submit as a hard copy, or submit on BlackBoard

Problem 1-5) Let x(t) be a function which is periodic in 2π

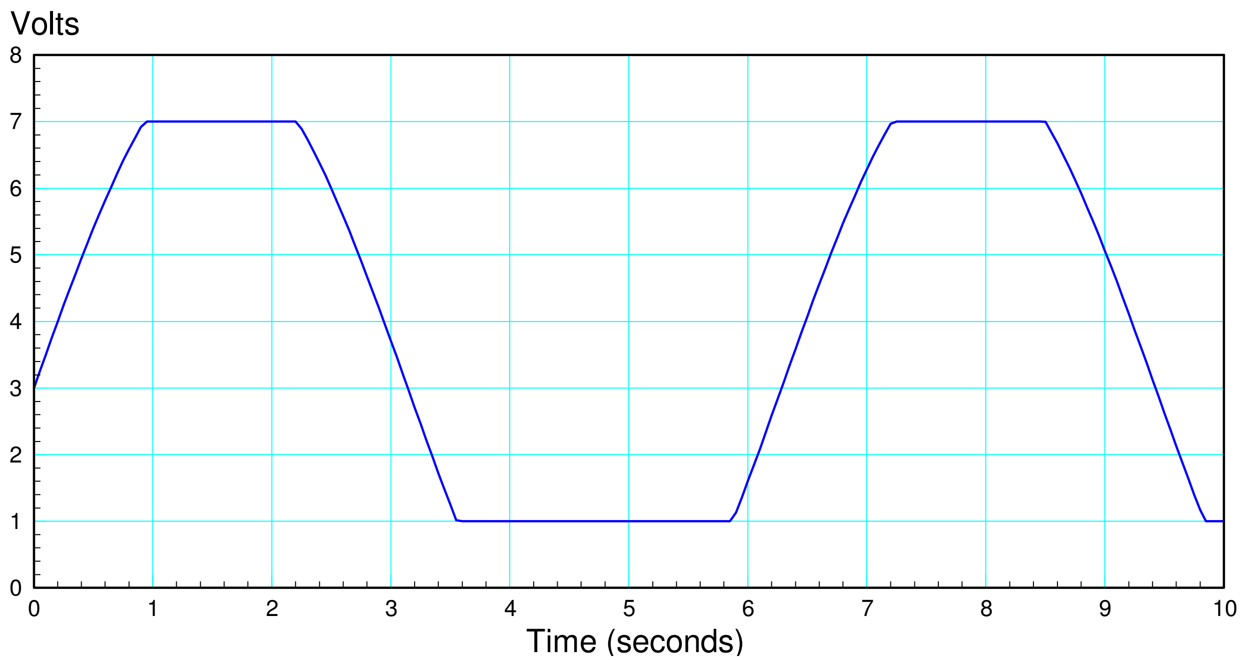
$$x(t) = x(t + 2\pi)$$

Over the interval $(0, 2\pi)$ x(t) is

$$x(t) = 5\sin(t) + 3$$

clipped at +7V and +1V. In Matlab:

```
t = [0:0.001:2*pi]';
x = 5*sin(t) + 3;
x = min(x, 7);
x = max(x, 1);
plot(t,x)
```



x(t) Note that x(t) repeats repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

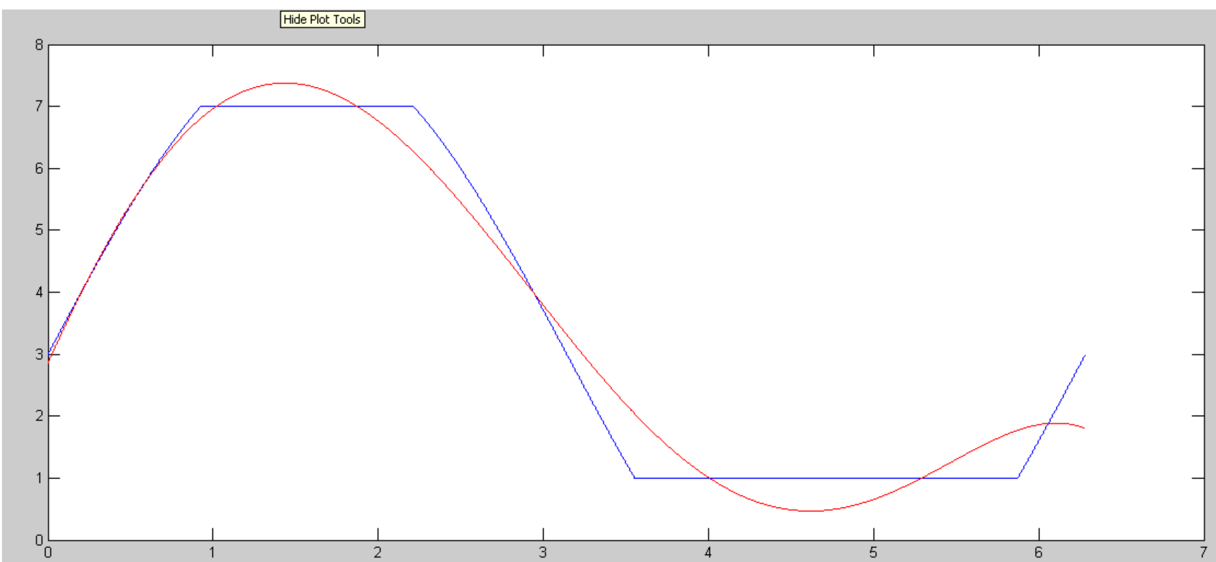
$$x(t) \approx a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]';
x = 5*sin(t) + 3;
x = min(x, 7);
x = max(x, 1);
plot(t,x)
>> B = [t.^0, t, t.^2, t.^3, t.^4, t.^5];
>> A = inv(B'*B)*B'*x

a0    2.8578
a1    5.9148
a2   -1.2585
a3   -0.7887
a4    0.2521
a5   -0.0190

>> plot(t,x,'b',t,B*A,'r')
>>
```



Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

Plot x(t) along with it's approximation.

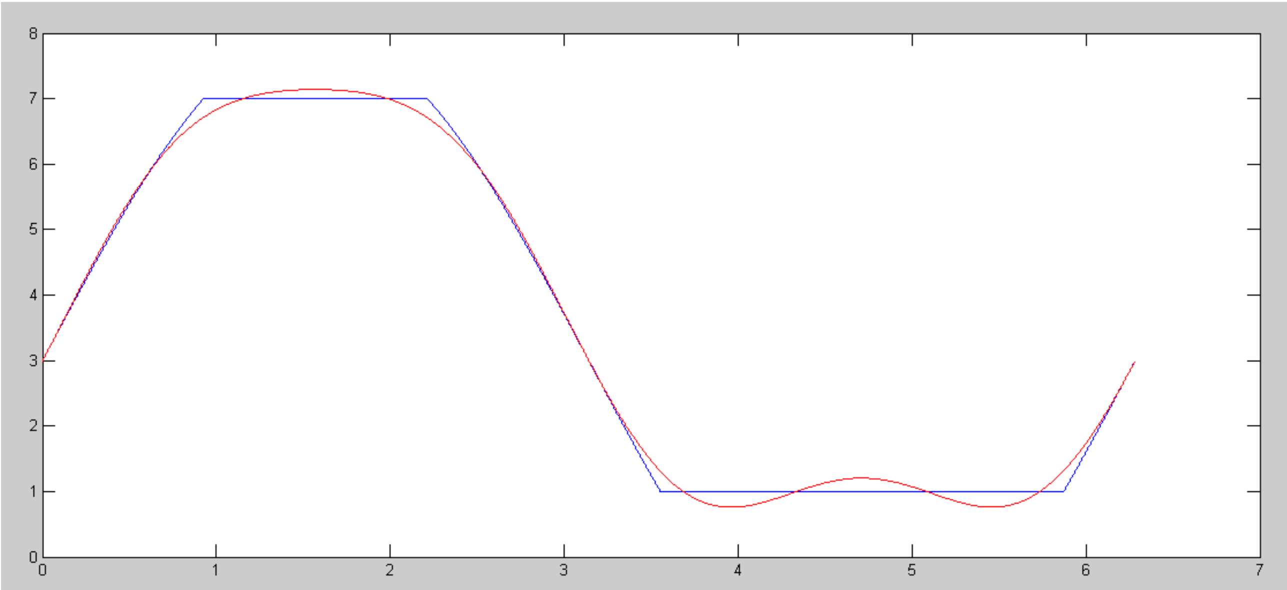
```
>> B = [t.^0, cos(t), sin(t), cos(2*t), sin(2*t), cos(3*t), sin(3*t)];
>> A = inv(B'*B)*B'*x

a0    3.5851
a1    0.0000
a2    3.4782
a3   -0.5877
a4   -0.0000
a5    0.0000
a6    0.5101

>> plot(t,x,'b',t,B*A,'r')
```

Note:

- This is just a curve fit with a different basis
- What's useful about this curve fit is the results are sine waves



3) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

```
>> a0 = mean(x)

a0 =    3.5850

>> a1 = 2*mean(x .* cos(t))

a1 =  7.7784e-004

>> b1 = 2*mean(x .* sin(t))

b1 =    3.4777

>> a2 = 2*mean(x .* cos(2*t))

a2 =   -0.5868

>> b2 = 2*mean(x .* sin(2*t))

b2 = -2.1011e-007

>> a3 = 2*mean(x .* cos(3*t))

a3 =  7.7778e-004

>> b3 = 2*mean(x .* sin(3*t))

b3 =    0.5100
```

Note:

- This is the same result as problem #2
- Fourier transforms is just curve fitting where the basis is a bunch of sine waves

Superposition:

Assume X and Y are related by

$$Y = \left(\frac{1.5}{s^3+1.7s^2+2.2s+1.2}\right)X$$

4) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_0$$

```
>> X0 = a0;
>> s = 0;
>> Y0 = (1.5 / (s^3 + 1.7*s^2 + 2.2*s + 1.2)) * X0

Y0 =    4.4812
```

$$y_0(t) = 4.4812$$

5) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_1\cos(t) + b_1\sin(t)$$

```
>> X1 = a1 - j*b1

X1 =   0.0008 - 3.4777i

>> s = j*1;
>> Y1 = (1.5 / (s^3 + 1.7*s^2 + 2.2*s + 1.2)) * X1

Y1 =  -3.7045 + 1.5426i
```

meaning

$$y_1(t) = -3.7045\cos(t) - 1.5426\sin(t)$$

6) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_2 \cos(2t) + b_2 \sin(2t)$$

```
>> X2 = a2 - j*b2

X2 =   -0.5868 + 0.0000i

>> s = j*2;
>> Y2 = (1.5 / (s^3 + 1.7*s^2 + 2.2*s + 1.2)) * X2

Y2 =    0.1112 - 0.0715i
```

meaning

$$y_2(t) = 0.1112 \cos(2t) + 0.0715 \sin(2t)$$

7) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_3 \cos(3t) + b_3 \sin(3t)$$

```
>> X3 = a3 - j*b3

X3 =   0.0008 - 0.5100i

>> s = j*3;
>> Y3 = (1.5 / (s^3 + 1.7*s^2 + 2.2*s + 1.2)) * X3

Y3 =   0.0254 + 0.0176i
```

meaning

$$y_3(t) = 0.0254 \cos(3t) - 0.0176 \sin(3t)$$

8) Plot y(t) when x(t) is the sum of x(t) for problems 4..7

The total answer is the sum of all four parts

$$y(t) = y_0 + y_1 + y_2 + y_3$$

$$y(t) = 4.4812 - 3.7045\cos(t) - 1.5426\sin(t)$$
$$+0.1112\cos(2t) + 0.0715\sin(2t)$$
$$0.0254\cos(3t) - 0.0176\sin(3t)$$

In Matlab

```
>> y0 = 4.4812;
>> y1 = real(Y1)*cos(t)-imag(Y1)*sin(t);
>> y2 = real(Y2)*cos(2*t) - imag(Y2)*sin(2*t);
>> y3 = real(Y3)*cos(3*t) - imag(Y3)*sin(3*t);
>>
>> y = y0+y1+y2+y3;
>> plot(t,x,'b',t,y,'r')
```