ECE 111 - Homework #14

ECE 343 Signals & Systems
Due Monday, November 27th

Filter Analysis

1) A filter has the following transfer function

$$Y = \left(\frac{10(s+2)}{(s+0.5)(s+6)(s+7)} \right) X$$

1a) What is the differential equation relating X and Y?

Cross multiply and multiply out

$$((s+0.5)(s+6)(s+7))Y = 10(s+2)X$$

$$(s^3 + 13.5s^2 + 48.5s + 21)Y = (10s + 20)X$$

Note that sY means *the derivative of y(t)*

$$y''' + 13.5y'' + 48.5y' + 21y = 10x' + 20x$$

1b) Find y(t) assuming $x(t) = 5$

At DC, s = 0

$$Y = \left(\frac{10(s+2)}{(s+0.5)(s+6)(s+7)} \right)_{s=0} \cdot (5)$$

$$Y = 4.762$$

1c) Find y(t) assuming $x(t) = 5\sin(2t)$

$$s = j2$$

$$X = 0 - j5$$

$$Y = \left(\frac{10(s+2)}{(s+0.5)(s+6)(s+7)} \right)_{s=j2} \cdot (0 - j5)$$

```
>> s = j*2;
>> X = 0 - j*5;
>> Y = 10*(s+2) / ( (s+0.5)*(s+6)*(s+7) ) * X

Y =  -1.3541 - 0.6215i
```

meaning

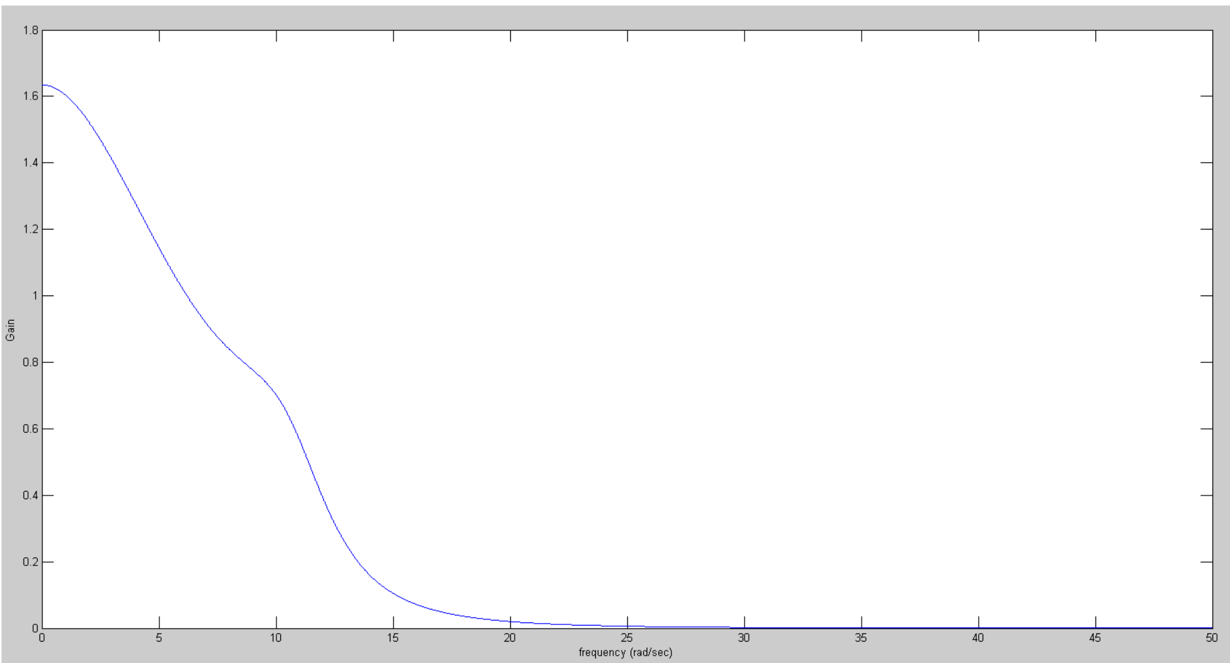
$$y(t) = -1.3541\cos(2t) + 0.6215\sin(2t)$$

2) Plot the gain vs. frequency for this filter from 0 to 50 rad/sec.

- Low-Pass Filter

$$Y = \left(\frac{50{,}000}{(s+4.8)\left(s^2+11.3s+51.8\right)\left(s^2+4.69s+123\right)} \right) X$$

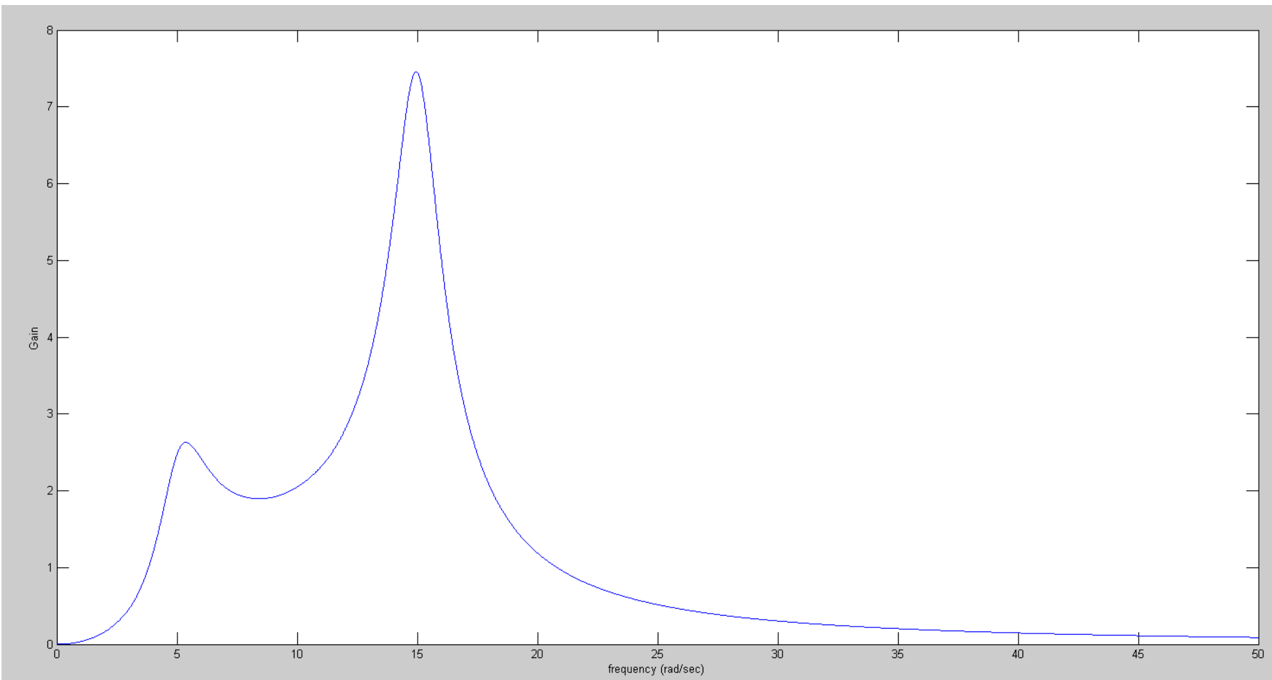
```
>> w = [0:0.01:50]';
>> s = j*w;
>> G2 = 50e3 ./ ( (s+4.8).*(s.^2 + 11.3*s + 51.8).*(s.^2 + 4.69*s + 123) );
>> plot(w,abs(G2))
>> xlabel('frequency (rad/sec)');
>> ylabel('Gain');
>>
```



3) Plot the gain vs. frequency for this filter from 0 to 50 rad/sec.

$$Y = \left(\frac{200 \cdot s^2}{(s+1 \pm j5)(s+1 \pm j15)} \right) X = \left(\frac{200 \cdot s^2}{\left(s^2+2s+26\right)\left(s^2+2s+226\right)} \right) X$$

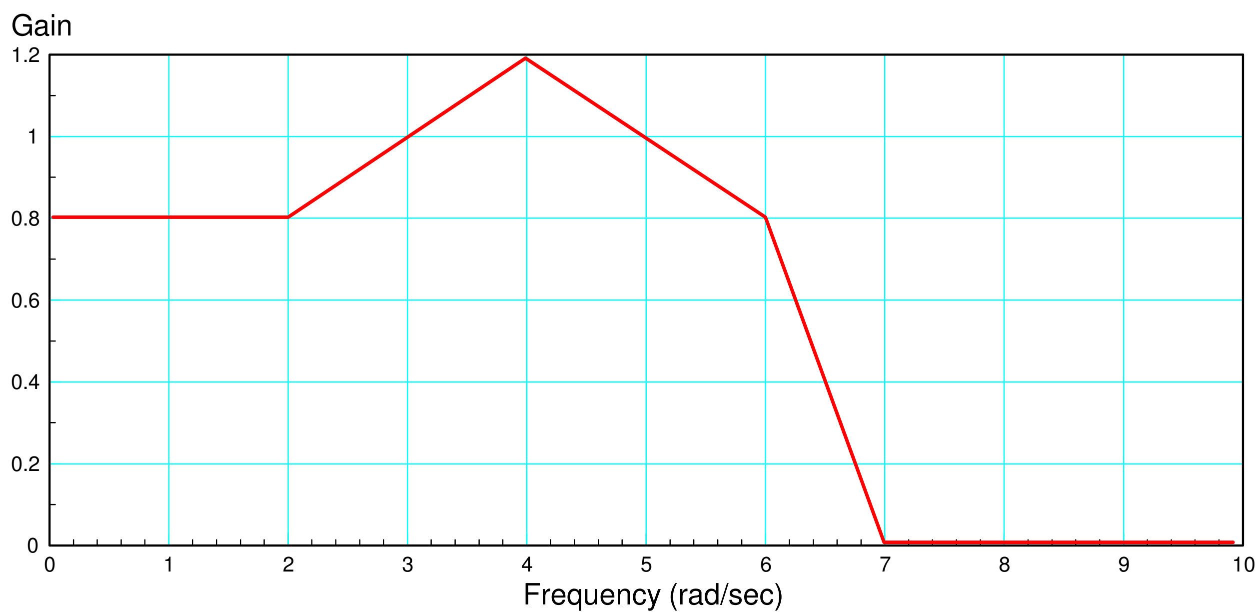
```
>> w = [0:0.01:50]';
>> s = j*w;
>> G = 200*s.^2 ./ ( (s+1+j*5).*(s+1-j*5).*(s+1+j*15).*(s+1-j*15) );
>> plot(w,abs(G))
>> xlabel('frequency (rad/sec)');
>> ylabel('Gain');
>>
```



Filter Design

4) Write an m-file, cost.m, which

- Is passed an array, z, with each element representing (a, b, c, d, e, f, g)
- Computes the gain, G(s) for this value of (a, b, c, d, e, f, g)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain



Step 1: Come up with a model for G(s). Use piecewise linear funcitons

$G = 0.8$ $\quad$ w < 2

$G = 0.2\omega + 0.4$ $\quad$ 2 < w < 4

$G = 2 - 0.2\omega$ $\quad$ 4 < w < 6

$G = 5.6 - 0.8w$ $\quad$ 6 < w < 7

$G = 0$ $\quad$ w > 7

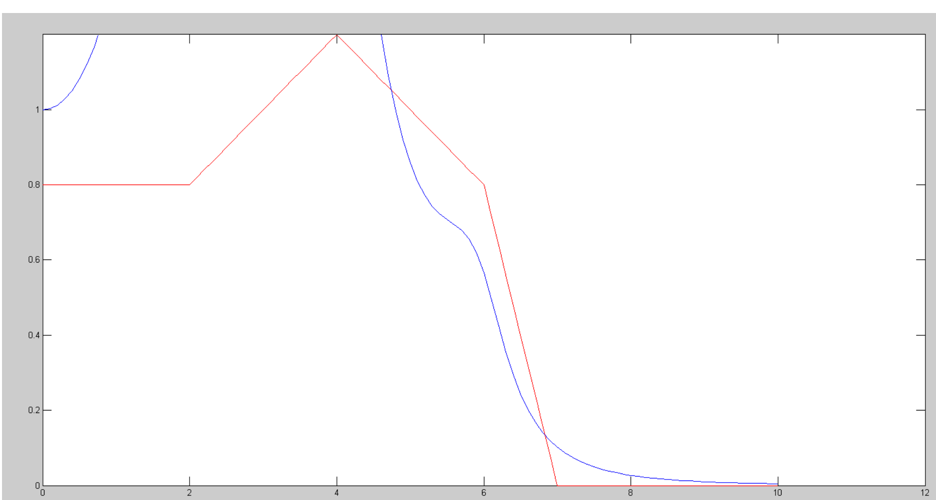
Code:

```
function [ J ] = costF( z )
    a = z(1);
    b = z(2);
    c = z(3);
    d = z(4);
    e = z(5);
    f = z(6);
    g = z(7);

    w = [0:0.1:10]' + 1e-6;
    s = j*w;

    G1 = 0.8 * (w < 2);
    G2 = (0.2*w+0.4).*(w>2).*(w<4);
    G3 = (2 - 0.2*w).*(w>4).*(w<6);
    G4 = (5.6 - 0.8*w).*(w>6).*(w<7);
    Gideal = G1 + G2 + G3 + G4;

    G = a ./ ( (s.^2 + b*s + c).*(s.^2 + d*s + e).*(s.^2+f*s+g) );

    e = abs(Gideal) - abs(G);
    J = sum(e .^ 2);

    plot(w,abs(Gideal),'r',w,abs(G),'b');
    ylim([0,1.2]);
    pause(0.01);

end
```

5) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)}\right) = \left(\frac{2304}{(s^2+s+4)(s^2+s+16)(s^2+s+36)}\right)$$

```
>> costF([2304,1,4,1,16,1,36])

ans =   288.4235
```



6) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right)$$

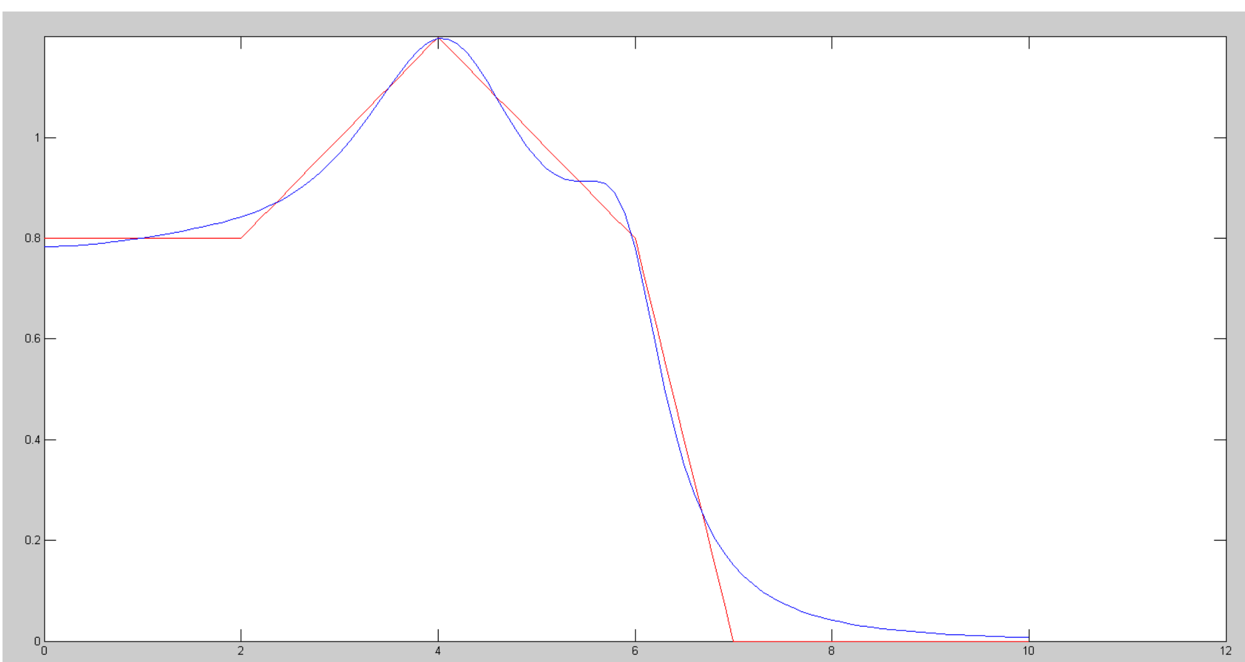
a) Give the resulting (a, b, c, d, e, f, g)

```
>> [Z,e] = fminsearch('costF',[2304,1,4,1,16,1,36])

Z =     4004.0    4.7245    7.8955    1.9387   18.0239    1.0002   35.8939

e =     0.1537
```

b) Give the resulting filter, and

$$G(s) = \left(\frac{4004}{(s^2+4.72s+7.89)(s^2+1.93s+18.02)(s^2+1.00s+35.89)} \right)$$

c) Plot the 'optimal' filter's gain vs. frequency



Note: You can design some pretty good filters using Matlab and fminsearch, even if you know nothing about filter design.