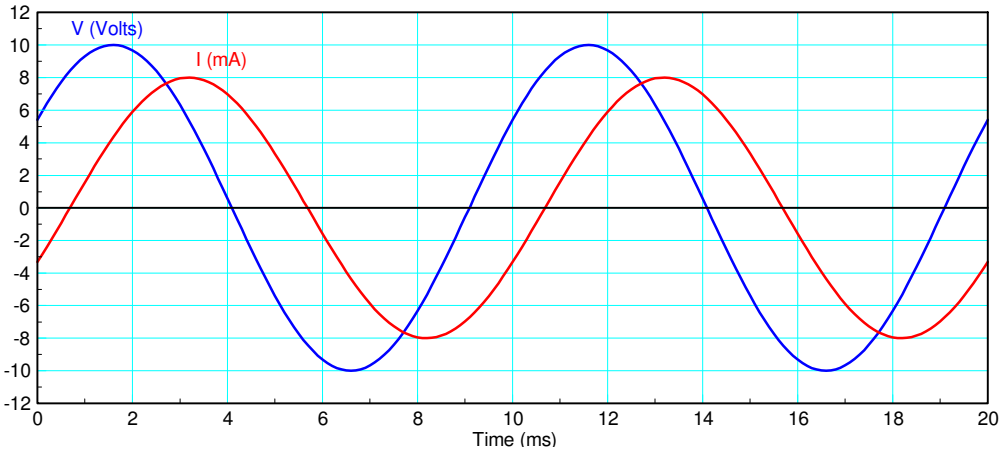
ECE 111 - Homework #13

ECE 311 Circuits II - Phasors
Due Monday, November 20th

Phasor Voltages

1) Express V and I as phasors (i.e. as complex numbers)

- From this, determine the impedance, Z = V/I



Voltage:

- Peak = 10V
- period = 10 ms
- delay = 1.5 ms

$$\theta = -\left(\frac{1.5s}{10s}\right)360^0 = -54.0^0$$

$$V = (10\angle -54.0^0)V$$

Current

- Peak = 8mA
- delay = 3.2ms

$$\theta = -\left(\frac{3.2ms}{10ms}\right)360^0 = -115.2^0$$

$$I = (8\angle -1115.2^0)mA$$

Impedance

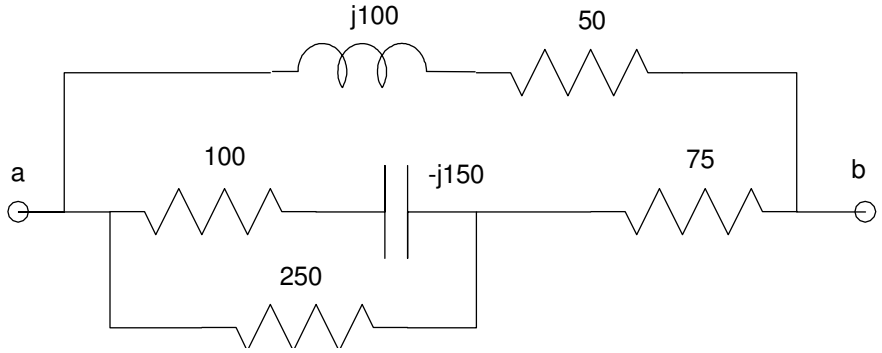
$$Z = \frac{V}{I} = \left(\frac{10\angle -54.0^0}{0.008\angle -115.2^0}\right)$$

$$Z = 1250\angle 61.2^0\Omega \qquad \textit{polar form}$$

$$Z = (602.2 + j1095.4)\Omega \qquad \textit{rectangular form}$$

Phasor Impedances

2) Determine the impedance, Zab



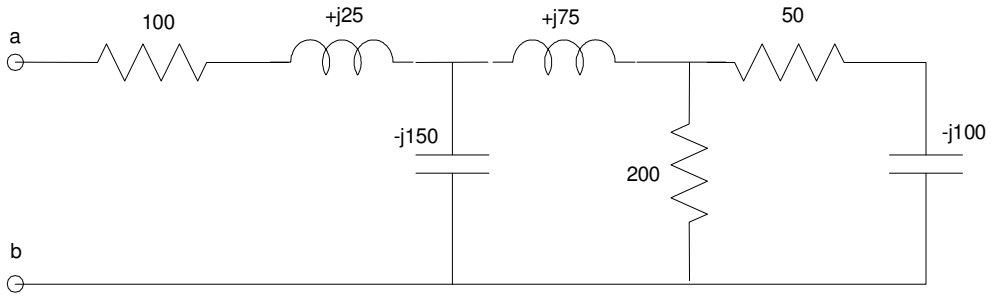
In Matlab

```
>> a = 100 - j*150;

>> b = 1/(1/a + 1/250)

b =  99.1379 -64.6552i

>> c = b + 75

c =  1.7414e+002 -6.4655e+001i

>> d = 1 / (1/c + 1/(50+j*100))

d =  75.7848 +51.3185i
```

With a HP42

```
100
enter
-150
complex
1/x
250
1/x
+
1/x
75
+
1/x
50
enter
100
complex
1/x
+
1/x
```

3) Determine the impedance, Zab



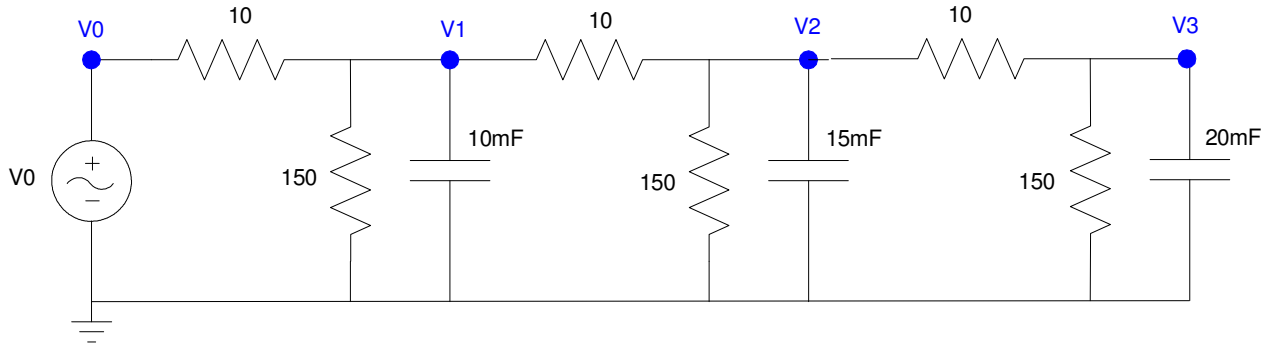
In Matlab

```
>> a = 50 - j*100;
>> b = 1 / (1/a + 1/200)

b =  62.0690 -55.1724i

>> c = b + j*75

c =  62.0690 +19.8276i

>> d = 1 / ( 1/c + 1/(-j*150) )

d =  67.1503 - 9.1710i

>> e = d + 100 + j*25

e =  1.6715e+002 +1.5829e+001i
```

With a HP42

```
50
enter
-100
complex
1/x
200
1/x
+
1/x
0
enter
75
complex
+
1/x
0
enter
-150
complex
1/x
+
1/x
100
enter
25
complex
+
```

Voltage Nodes with Phasors



4) Assume V0 = 10.

a) Determine the impedances of each element at 0 rad/sec

$$V_0 = 10$$

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

Ignore capacitors at DC (the current is zero(

b) Write the votlage node equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1 - V_2}{10}\right) = 0$$

$$\left(\frac{V_2 - V_1}{10}\right) + \left(\frac{V_2}{150}\right) + \left(\frac{V_2 - V_3}{10}\right) = 0$$

$$\left(\frac{V_3 - V_2}{10}\right) + \left(\frac{V_3}{150}\right) = 0$$

c) Solve for V1, V2, and V3.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{10}\right)V_0 + \left(\frac{1}{10} + \frac{1}{150} + \frac{1}{10}\right)V_1 - \left(\frac{1}{10}\right)V_2 = 0$$

$$-\left(\frac{1}{10}\right)V_1 + \left(\frac{1}{10} + \frac{1}{150} + \frac{1}{10}\right)V_2 - \left(\frac{1}{10}\right)V_3 = 0$$

$$-\left(\frac{1}{10}\right)V_2 + \left(\frac{1}{10} + \frac{1}{150}\right)V_3 = 0$$

Place in matrix form

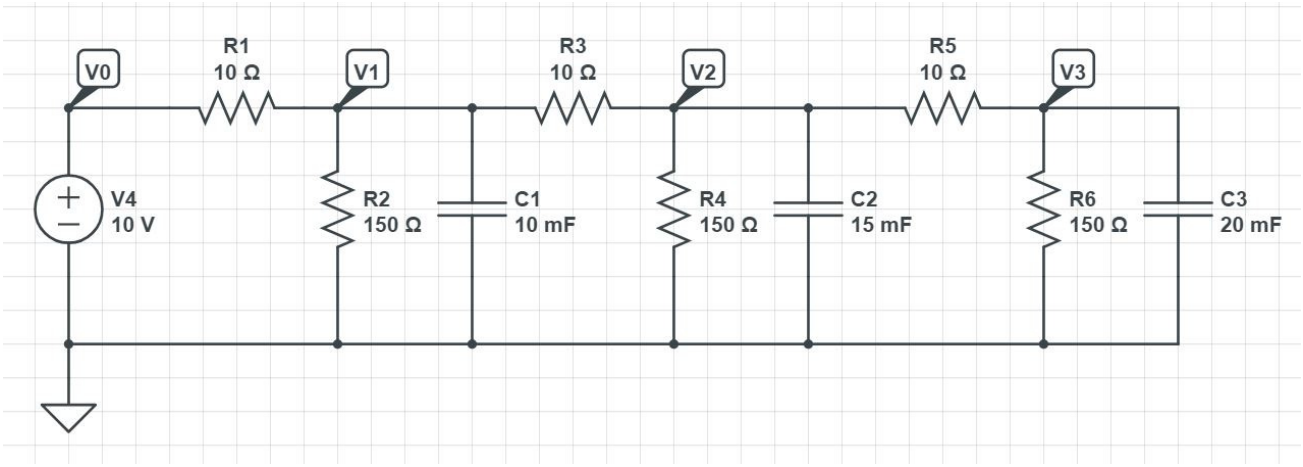
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{10}\right) & \left(\frac{-1}{10}\right) & 0 \\ 0 & \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{10}\right) & \left(\frac{-1}{10}\right) \\ 0 & 0 & \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{150}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

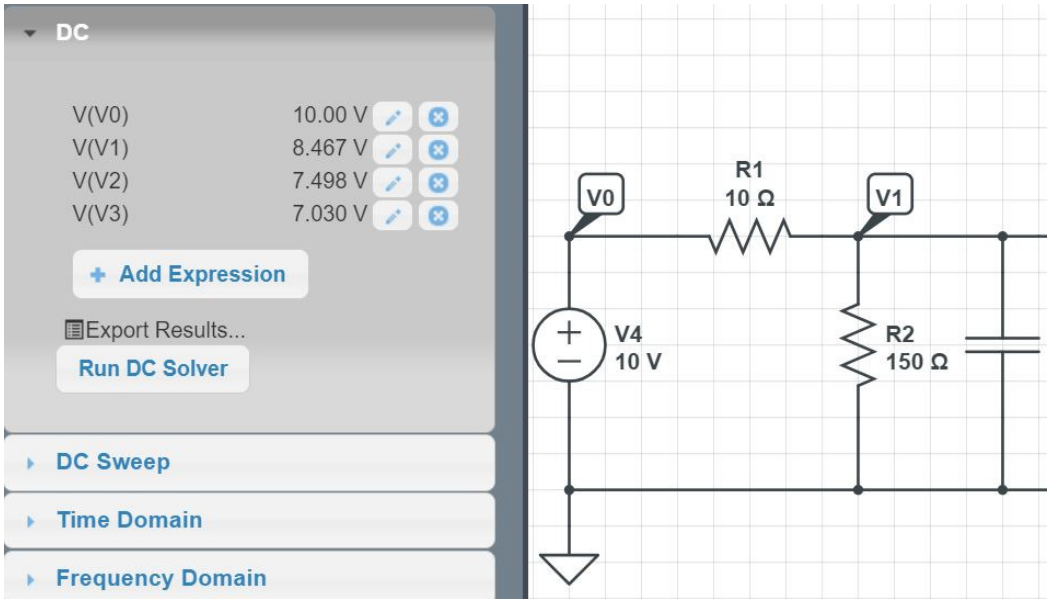
```
>> a1 = [1,0,0,0];
>> a2 = [-1/10,1/10+1/150+1/10,-1/10,0];
>> a3 = [0,-1/10,1/10+1/150+1/10,-1/10];
>> a4 = [0,0,-1/10,1/10+1/150];
>> A = [a1;a2;a3;a4]

    1.0000         0         0         0
   -0.1000    0.2067   -0.1000         0
         0   -0.1000    0.2067   -0.1000
         0         0   -0.1000    0.1067

>> B = [10;0;0;0]

    10
     0
     0
     0

>> V = inv(A)*B

V0   10.0000
V1    8.4670
V2    7.4984
V3    7.0298

>>
```

5) Check your results in CircuitLab

Input the circuit in CircuitLab

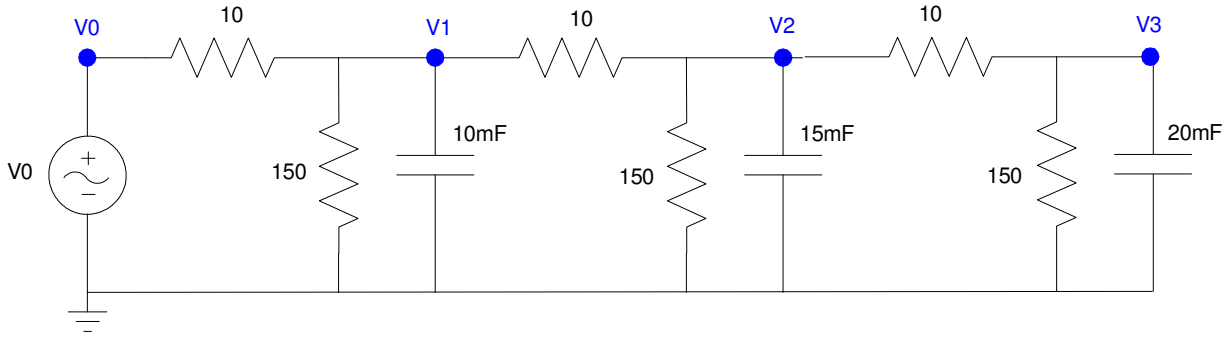


Run a DC simulation



The answers are the same as Matlab

6) Assume $V_0 = 10\sin(2t)$ *10V, 2 rad/sec sine wave (0.318Hz)*



a) Determine the impedances of each element at 2 rad/sec

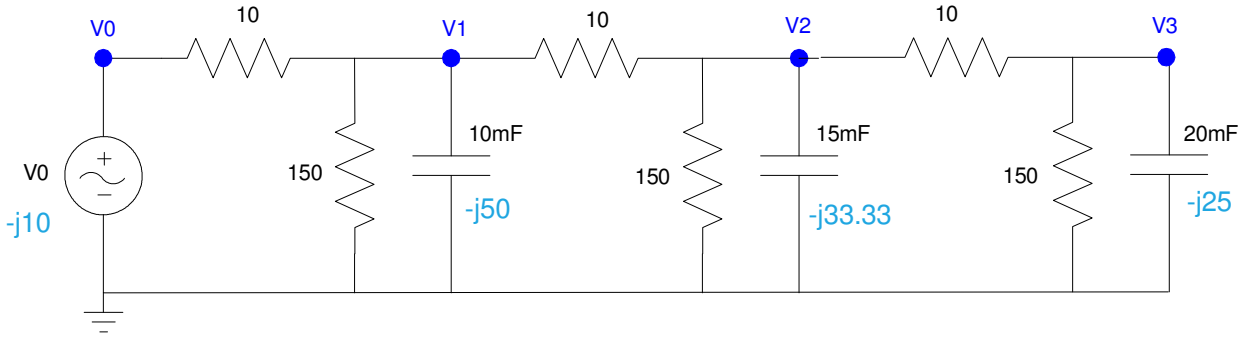
$$V_0 = 0 - j10, \qquad \omega = 2$$

$$C_1 \rightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.01)} = -j50$$

$$C_2 \rightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.015)} = -j33.3$$

$$C_1 \rightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.02)} = -j25$$

Redraw the circuit



b) Write the votlage node equations

$$V_0 = -j10$$

$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1}{-j50}\right) + \left(\frac{V_1 - V_2}{10}\right) = 0$$

$$\left(\frac{V_2 - V_1}{10}\right) + \left(\frac{V_2}{150}\right) + \left(\frac{V_2}{-j33.33}\right) + \left(\frac{V_2 - V_3}{10}\right) = 0$$

$$\left(\frac{V_3 - V_2}{10}\right) + \left(\frac{V_3}{150}\right) + \left(\frac{V_3}{-j25}\right) = 0$$

c) Solve for V1, V2, and V3 as complex numbers

Group terms

$$V_0 = -j10$$

$$-\left(\frac{1}{10}\right)V_0 + \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{-j50}+\frac{1}{10}\right)V_1 - \left(\frac{1}{10}\right)V_2 = 0$$

$$-\left(\frac{1}{10}\right)V_1 + \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{-j33.33}+\frac{1}{10}\right)V_2 - \left(\frac{1}{10}\right)V_3 = 0$$

$$-\left(\frac{1}{10}\right)V_2 + \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{-j25}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{-j50}+\frac{1}{10}\right) & \left(\frac{-1}{10}\right) & 0 \\ 0 & \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{-j33.3}+\frac{1}{10}\right) & \left(\frac{-1}{10}\right) \\ 0 & 0 & \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{-j25}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> a1 = [1,0,0,0];
>> a2 = [-1/10,1/10+1/150+1/10+1/(-j*50),-1/10,0];
>> a3 = [0,-1/10,1/10+1/150+1/10+1/(-j*33.33),-1/10];
>> a4 = [0,0,-1/10,1/10+1/150+1/(-j*25)];
>> A = [a1;a2;a3;a4]

   1.0000                  0                  0                  0
  -0.1000            0.2067 + 0.0200i  -0.1000                  0
        0           -0.1000             0.2067 + 0.0300i  -0.1000
        0                 0            -0.1000             0.1067 + 0.0400i

>> B = [-j*10;0;0;0]

        0 -10.0000i
        0
        0
        0

>> V = inv(A)*B

V0        0 -10.0000i
V1  -2.2688 - 6.2245i
V2  -3.4439 - 3.3177i
V3  -3.8532 - 1.6654i
```

d) Express V1, V2, and V3 in terms of sine and cosine function:

```
V0         0 -10.0000i
V1   -2.2688 - 6.2245i
V2   -3.4439 - 3.3177i
V3   -3.8532 - 1.6654i
```

meaning

$$V_0 = 10\sin(2t)$$

$$V_1 = -2.2688\cos(2t) + 6.2245\sin(2t)$$

$$V_2 = -3.4439\cos(2t) + 3.3177\sin(2t)$$

$$V_3 = -3.8532\cos(2t) + 1.6654\sin(2t)$$

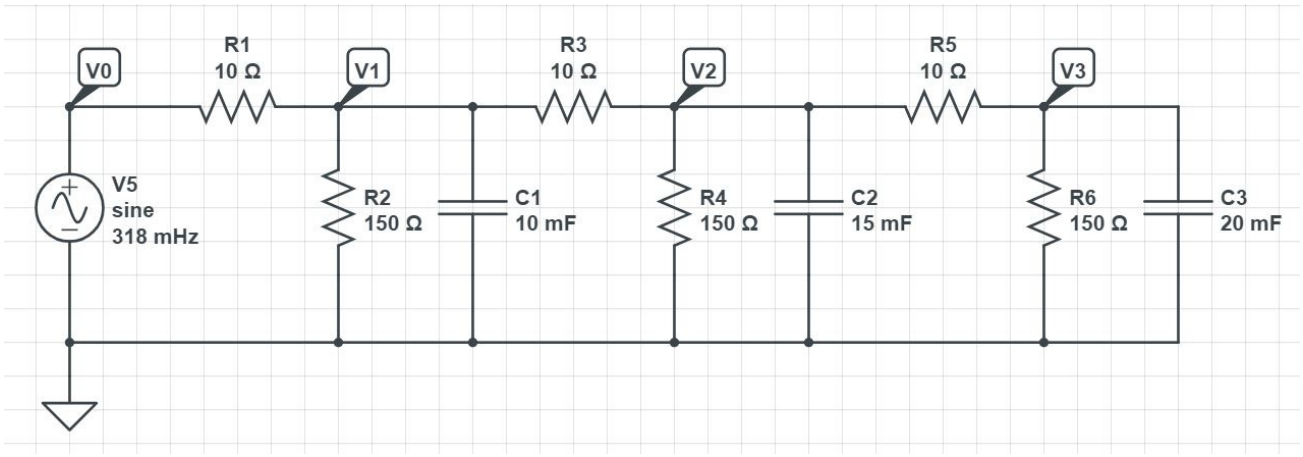
7) Check your results in CircuitLab using a transient simulation for 15 seconds (time step = 15ms).

Note: polar form works better for graphs. This tells you the magnitude of each voltage and its delay

```
>> abs(V)

   10.0000
    6.6251
    4.7820
    4.1977

>> angle(V)*180/pi

  -90.0000
 -110.0263
 -136.0689
 -156.6249

>>
```

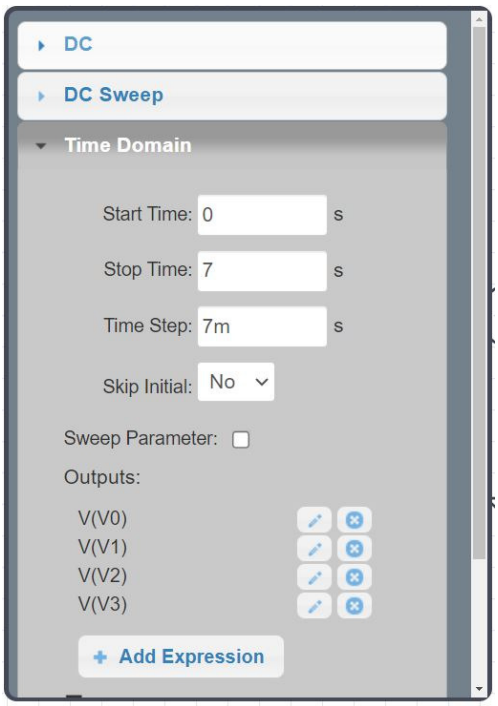
Input the system into CircuitLab. Note

$$2\frac{rad}{\sec} = \left(\frac{2}{2\pi}\right)Hz = 0.318Hz$$

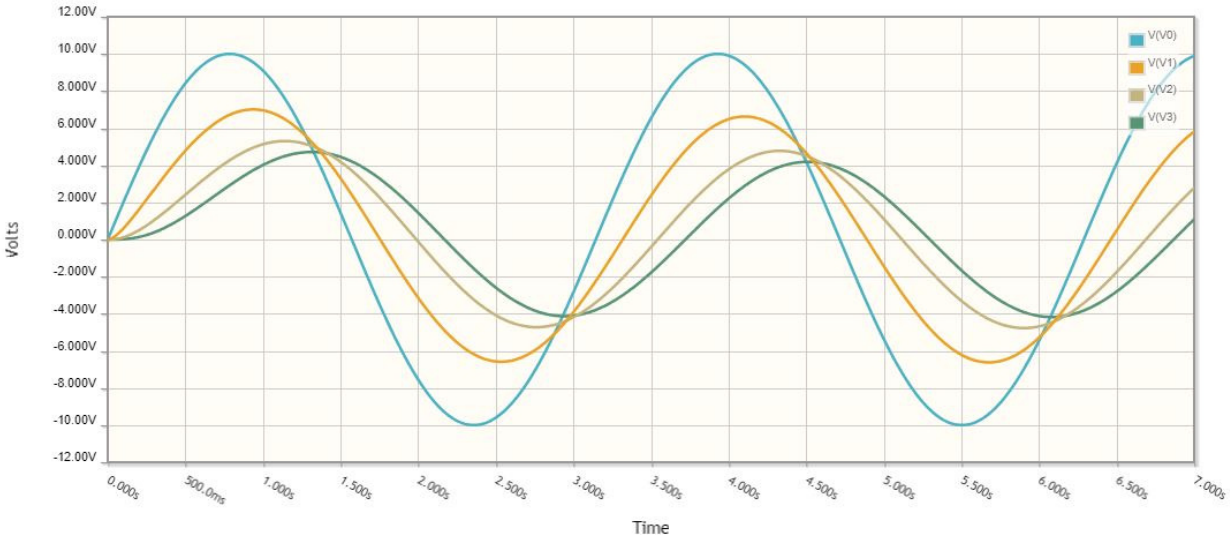


One cycle is 1/0.318 = 3 seconds. Simulate for two cycles (7 seconds)

- Make the step size 1000x smaller to get 1000 points on the plot



This results in



Note: This matches Matlab's numbers. The magnitude of the voltages should be

```
V0 = 10.0000V
V1 =  6.6251V
V2 =  4.7820V
V3 =  4.1977V
```