

ECE 111 - Homework #12

Complex Numbers
Due Monday, November 13th

Complex Numbers

1) Determine the rectangular or polar form of each complex number

- $7 + j9 = 11.402 \angle 52.125^\circ$
- $2 - j3 = 3.606 \angle -56.310^\circ$
- $4 \angle 50^\circ = 2.571 + j3.064$
- $9 \angle -22^\circ = 8.345 - j3.371$

2) Determine y as a complex number

$$y = (2 + j3) + (7 - j6) + (-2 + j12)$$

$$y = 7 + j9$$

Add the real parts and add the complex parts. In Matlab:

```
>> y = (2+j*3) + (7-j*6) + (-2+j*12)
y = 7.0000 + 9.0000i
```

$$y = (7 \angle 20^\circ) + (8 \angle -63^\circ) + (2 \angle 79^\circ)$$

$$y = 10.591 - j2.771$$

In Matlab

```
>> y = 7*exp(j*20*pi/180) + 8*exp(-j*63*pi/180) + 2*exp(j*79*pi/180)
y = 10.5914 - 2.7707i
```

3) Determine y as a complex number

$$y = \left(\frac{(2+j12)(9-j3)}{(7-j6)} \right)$$

```
>> y = (2+j*12) * (3-j*3) / (7-j*6)
```

```
y = 1.3412 + 5.4353i
```

$$y = \left(\left(\frac{2+j12}{7-j6} \right) + \left(\frac{9-j3}{7+j6} \right) \right) \left(\frac{4+j2}{8+j3} \right)$$

```
>> a = (2+j*12) / (7-j*6);
```

```
>> b = (9-j*3) / (7+j*6);
```

```
>> c = (4+j*2) / (8+j*3);
```

```
>> y = (a + b) * c
```

```
y = -0.0932 + 0.1202i
```

4) Determine y as a complex number

$$y = e^{(2+j3)}$$

```
>> y = exp(2 + j*3)
```

```
y = -7.3151 + 1.0427i
```

$$y = \ln(2 + j3)$$

```
>> y = log(2 + j*3)
```

```
y = 1.2825 + 0.9828i
```

$$y = (2 + j3)^{(4+j5)}$$

```
>> y = (2 + j*3)^(4+j*5)
```

```
y = -0.7530 - 0.9864i
```

note: Matlab works with complex numbers just as easily as it works with real numbers. The result tends to be a complex number, but if that doesn't bother you, it's just a number.

Partial Fractions with Complex Numbers

5) Determine the partial fraction expansion

$$\left(\frac{10(x+1)(x+2)}{(x+1+j2)(x+1-j2)(x+5)} \right) = \left(\frac{a}{x+1+j2} \right) + \left(\frac{b}{x+1-j2} \right) + \left(\frac{c}{x+5} \right)$$

```
>> x = -1-j*2;
>> a = 10*(x+1)*(x+2) / ( (x+1-j*2)*(x+5) )

a = 2.0000 - 1.5000i

>> x = -1+j*2;
>> b = 10*(x+1)*(x+2) / ( (x+1+j*2)*(x+5) )

b = 2.0000 + 1.5000i

>> x = -5;
>> c = 10*(x+1)*(x+2) / ( (x+1+j*2)*(x+1-j*2) )

c = 6
```

6) Determine the partial fraction expansion

$$\left(\frac{(x+j)(x-j)}{x(x+3)(x+2+j5)(x+2-j5)} \right) = \left(\frac{a}{x+0} \right) + \left(\frac{b}{x+3} \right) + \left(\frac{c}{x+2+j5} \right) + \left(\frac{d}{x+2-j5} \right)$$

```
>> x = 0;
>> a = (x+j)*(x-j) / ( (x+3)*(x+2+j*5)*(x+2-j*5) )

a = 0.0115

>> x = -3;
>> b = (x+j)*(x-j) / ( (x)*(x+2+j*5)*(x+2-j*5) )

b = -0.1282

>> x = -2-j*5;
>> c = (x+j)*(x-j) / ( (x)*(x+3)*(x+2-j*5) )

c = 0.0584 + 0.0849i

>> x = -2+j*5;
>> d = (x+j)*(x-j) / ( (x)*(x+3)*(x+2+j*5) )

d = 0.0584 - 0.0849i

>>
```

note: Matlab can do partial fraction expansion with complex numbers as well. Matlab doesn't really care: real or complex both work the same with Matlab.