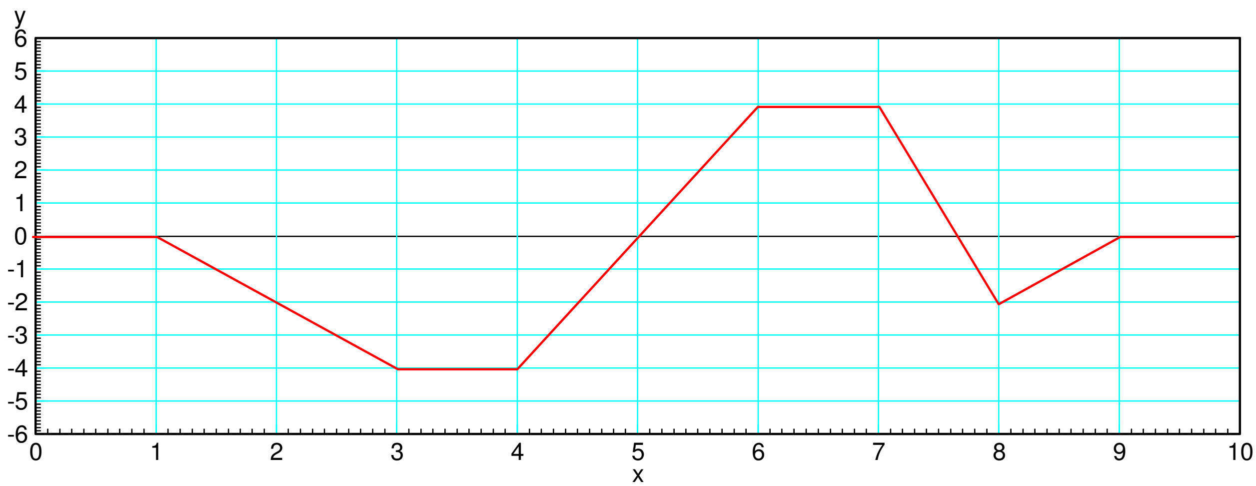
ECE 111 - Homework #11

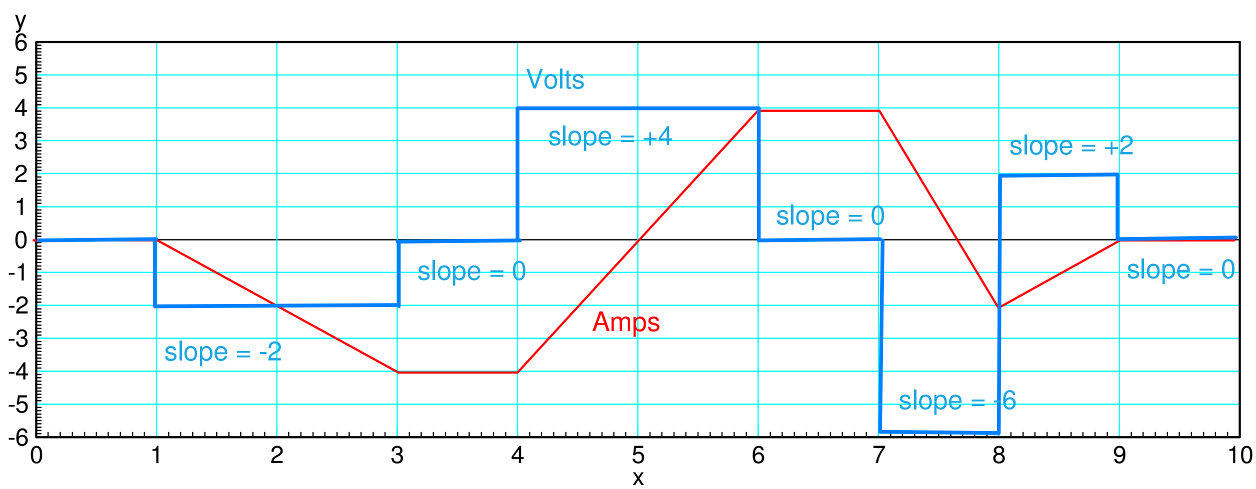
ECE 351 Electromagnetics - Wave Equation
Due Tuesday, November 7th
Please submit BlackBoard

1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

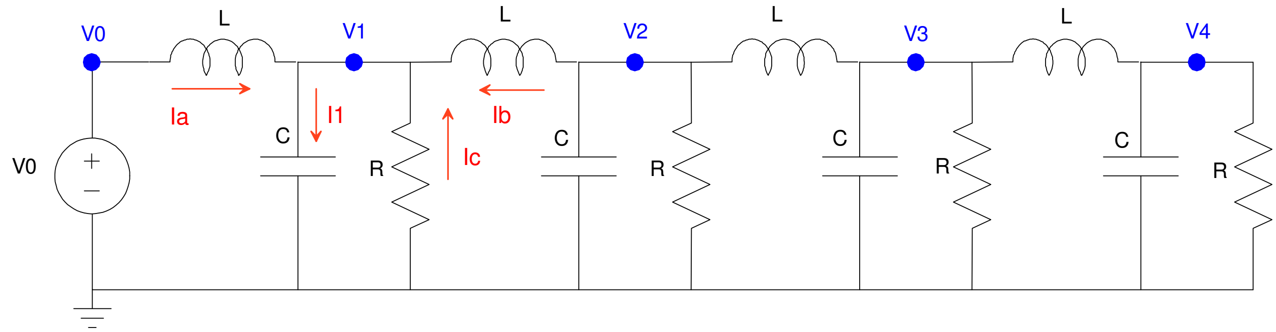
$$V = L\frac{dI}{dt}$$



With L = 1, the voltage is the derivative of the current (the slope)



4-Node RLC Circuit



R = 300Ω, C = 0.2F, L = 0.2H. Repeat for 30 nodes for problems 4-6

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)

From before, the current to a capacitor is C * dV/dt

$$I_1 = CV_1' = I_a + I_b + I_c$$

Take the derivative

$$CV_1'' = I_a' + I_b' + I_c'$$

For inductors, V = L dI/dt

$$V_0 - V_1 = LI_a'$$

$$V_2 - V_1 = LI_b'$$

Substitute

$$CV_1'' = \left(\frac{V_0 - V_1}{L}\right) + \left(\frac{V_2 - V_1}{L}\right) + \left(\frac{0 - V_1}{R}\right)'$$

Group terms and divide by C

$$V_1'' = \left(\frac{1}{LC}\right)V_0 - \left(\frac{2}{LC}\right)V_1 + \left(\frac{1}{LC}\right)V_2 - \left(\frac{1}{RC}\right)V_1'$$

Plug in numbers and you get the dynamics for nodes 1..3

$$V_1'' = 25V_0 - 50V_1 + 25V_2 - 0.0167V_1'$$

$$V_2'' = 25V_1 - 50V_2 + 25V_3 - 0.0167V_2'$$

$$V_3'' = 25V_2 - 50V_3 + 25V_4 - 0.0167V_3'$$

Node #4 is a little different since only inductor is connected

$$V_4'' = 25V_3 - 25V_4 - 0.0167V_4'$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*

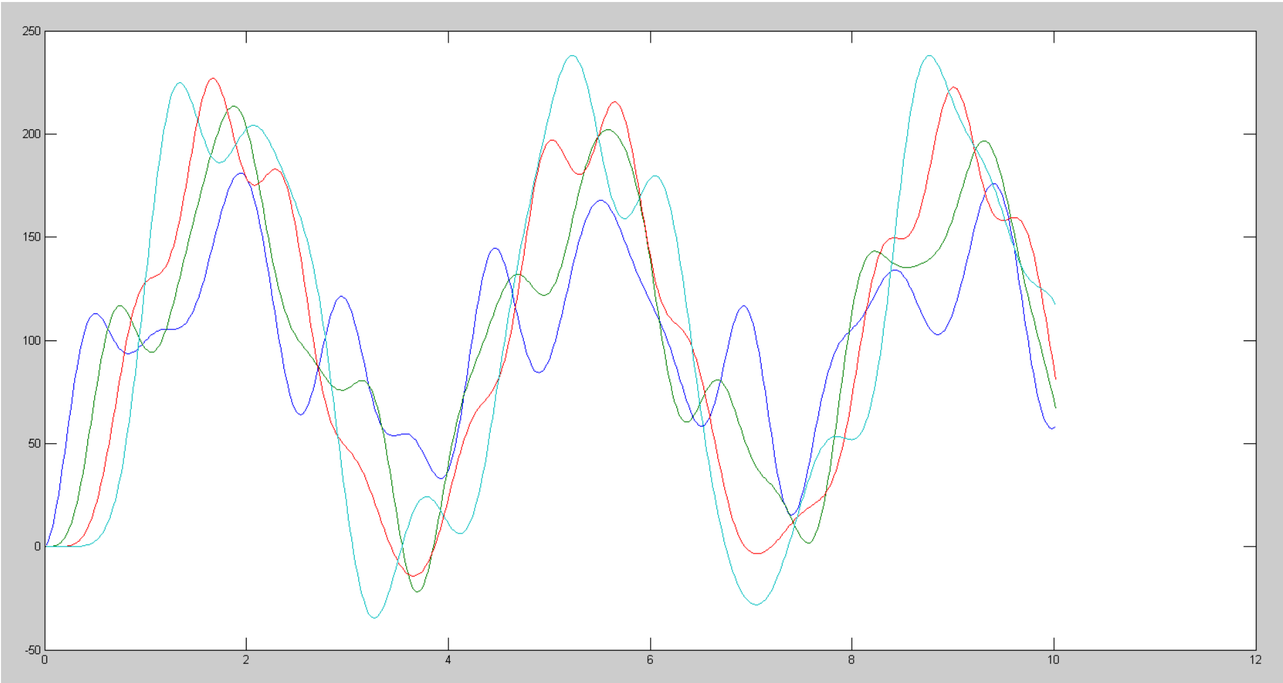
Code:

- Compute the second devative of each voltage
- Integrate once to get V'
- Integrate again to get V

```
V0 = 100;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;

dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;

V = [];

t = 0;
dt = 0.02;

while(t < 10)

   ddV1 = 25*V0 - 50*V1 + 25*V2 - 0.0167*dV1;
   ddV2 = 25*V1 - 50*V2 + 25*V3 - 0.0167*dV2;
   ddV3 = 25*V2 - 50*V3 + 25*V4 - 0.0167*dV3;
   ddV4 = 25*V3 - 25*V4         - 0.0167*dV4;

   dV1 = dV1 + ddV1*dt;
   dV2 = dV2 + ddV2*dt;
   dV3 = dV3 + ddV3*dt;
   dV4 = dV4 + ddV4*dt;

   V1 = V1 + dV1*dt;
   V2 = V2 + dV2*dt;
   V3 = V3 + dV3*dt;
   V4 = V4 + dV4*dt;

   t = t + dt;

   plot([0,1,2,3,4],[V0,V1,V2,V3,V4],'.-');
   ylim([-300,300]);
   clc
   disp(t)
   pause(0.01);

   V = [V; V1, V2, V3, V4];

   end

t = [1:length(V)]' * dt;
plot(t,V);
```

Result:

- Note: The wave equation behaves very differently than the heat equation



30-Node RLC Circuit (hint: modify the program Wave.m)

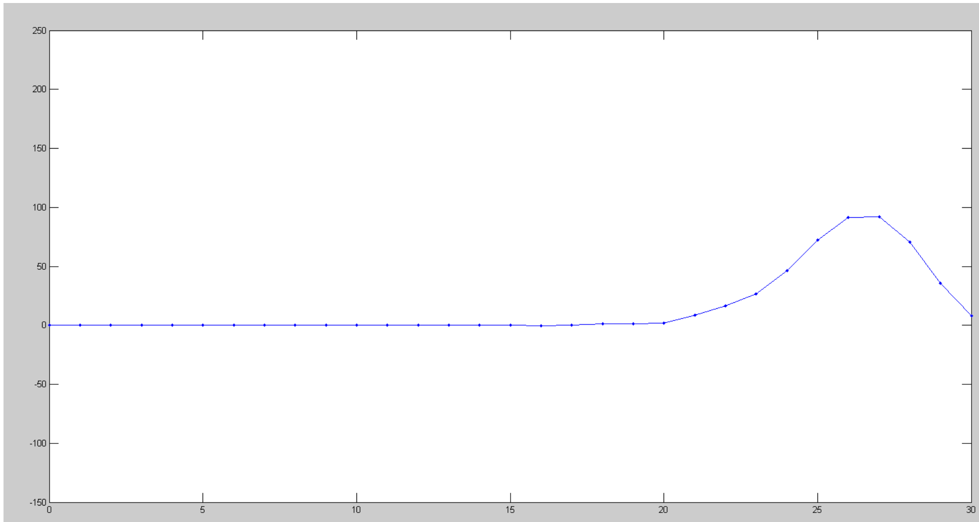
4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 12 seconds (just after the reflection) for $1 / R_{30}C = 0.01$

Option 1: Make 30 copies of each section of code

Option 2: Use for-loops

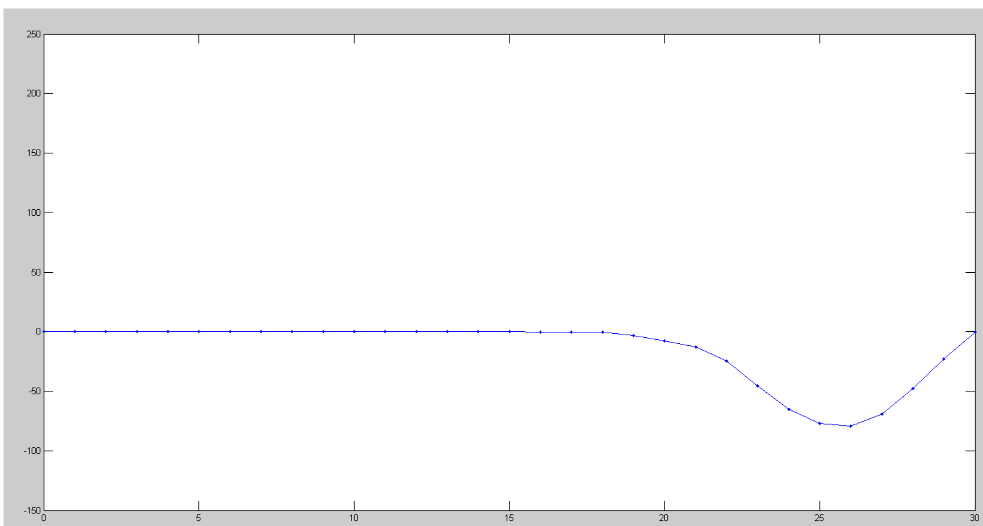
Code:

```
N = 30;   % number of nodes

V = zeros(N,1);
dV = zeros(N,1);

t = 0;
dt = 0.02;

while(t < 8)

   if (t < 2) V0 = 100 * ( ( sin(0.5*pi*t) )^2 );
      else V0 = 0;
      end

   ddV(1) = 25*V0 - 50*V(1) + 25*V(2) - 0.0167*dV(1);

   for i=2:N-1
      ddV(i) = 25*V(i-1) - 50*V(i) + 25*V(i+1) - 0.0167*dV(i);
      end

   ddV(N) = 25*V(N-1) - 25*V(N) - 0.0167*dV(N);
%                                  ^^^
%                               change this term


   for i=1:N
      dV(i) = dV(i) + ddV(i)*dt;
      V(i) =  V(i) +  dV(i)*dt;
      end
   t = t + dt;

   plot([0:N],[V0;V],'.-');
   ylim([-150,250]);
   clc
   disp(t)
   pause(0.01);

   end
```



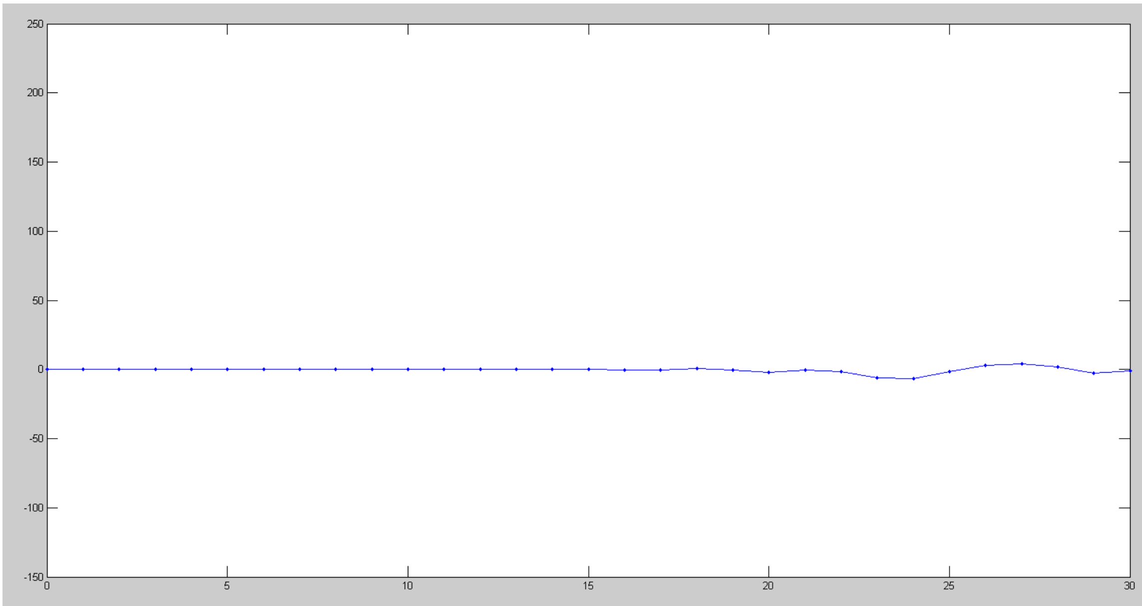
Result with 1/R30C = 0.01

5) Plot the voltage at t = 8 seconds for $1 / R_{30}C = 100$



Result with 1/R30 C = 100

6) Determine experimentally R_{30} so that the reflection is almost zero



Result with 1 / R30 C = 5