ECE 111 - Homework #9

Week #9: ECE 311 Circuits II - Due 11am, Tuesday, October 25th

1) Solve for Y

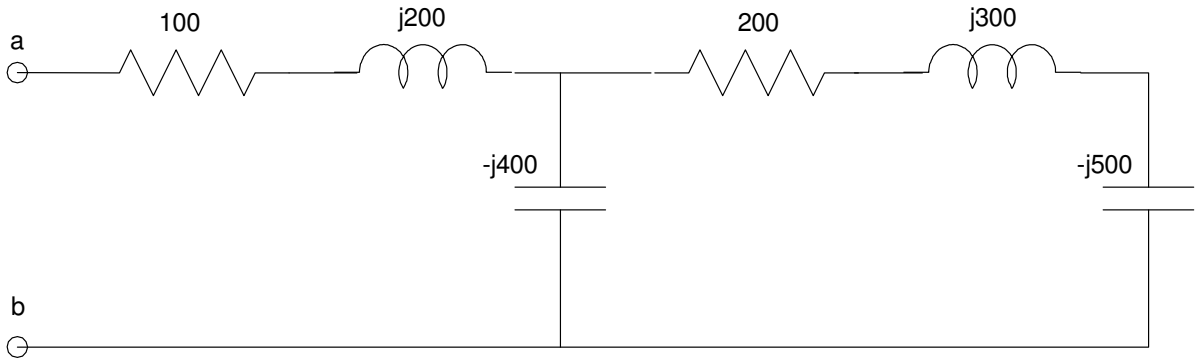
$$Y = \left(\frac{(5+j2)(7-j3)}{(2-j6)} \right)$$

```
>> Y = (5 + j*2)*(7 - j*3) / (2 - j*6)

Y =   2.2000 + 6.1000i
```

$$Y = \left(\left(\frac{5+j2}{2-j6} \right) + \left(\frac{7-j3}{2+j5} \right) \right) \left(\frac{8+j3}{7+j9} \right)$$

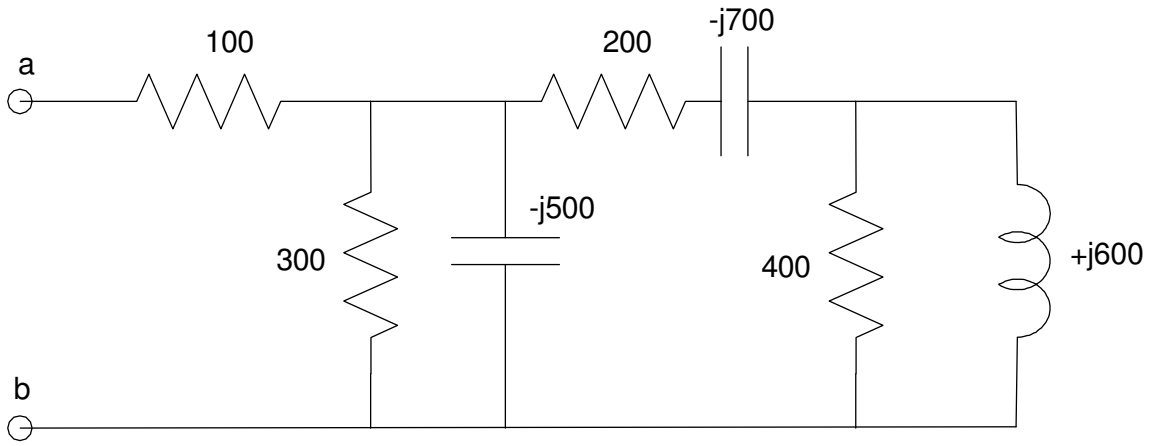
```
>> A = (5 + j*2)/(2 - j*6);
>> B = (7 - j*3)/(2 + j*5);
>> C = (8 + j*3) / (7 + j*9);
>> Y = (A + B)*C

Y =  -0.2751 - 0.3268i
```

2) Determine the impedance Zab



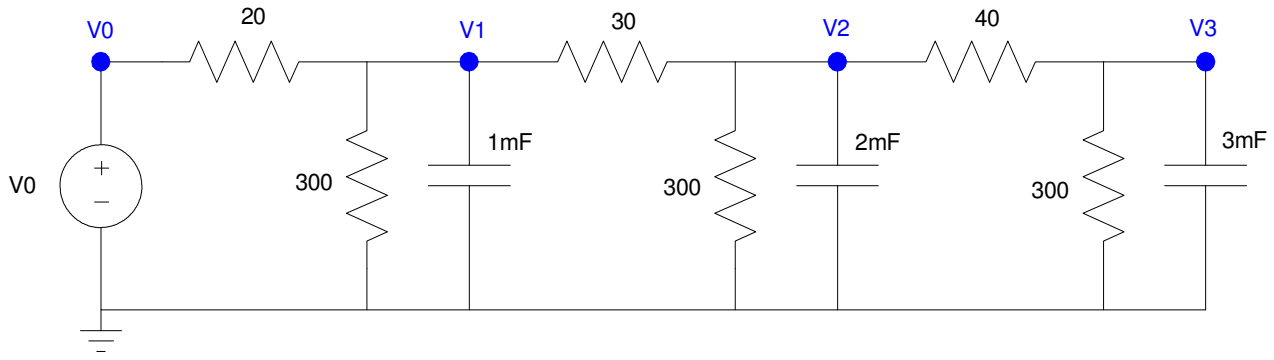

```
>> R1 = 200 + j*300 - j*500

R1 =  200.00 -200.00i

>> R2 = -j*400;
>> R3 = 1 / (1/R1 + 1/R2)

R3 =  80.000 -160.00i

>> Rab = R3 + 100 + j*200

Rab =  180.00 + 40.000i
```

3) Determine the impeadance Zab




```
>> R1 = 400;
>> R2 = j*600;
>> R3 = 1/( 1/R1 + 1/R2 )

R3 =  276.92 + 184.62i

>> R4 = R3 + (200 - j*700)

R4 =  476.92 - 515.38i

>> R5 = 300;
>> R6 = -j*500;
>> R7 = 1/ (1/R4 + 1/R5 + 1/R6)

R7 =  154.87 - 109.67i

>> Rab = R7 + 100

Rab =  254.87 - 109.67i

>>
```

4) Assume $V_0 = 10$



a) Determine the impedances of each element at 0 rad/sec

$$R \rightarrow R$$

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

b) Write the votlage node equations

The current is zero for the capacitors - meaning you could ignore them if you want

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{20}\right) + \left(\frac{V_1}{300}\right) + 0 + \left(\frac{V_1 - V_2}{30}\right) = 0$$

$$\left(\frac{V_2 - V_1}{30}\right) + \left(\frac{V_2}{300}\right) + 0 + \left(\frac{V_2 - V_3}{40}\right) = 0$$

$$\left(\frac{V_3 - V_2}{40}\right) + \left(\frac{V_4}{300}\right) + 0 = 0$$

c) Solve for V1, V2, and V3.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{20}\right)V_0 + \left(\frac{1}{20}+\frac{1}{300}+\frac{1}{30}\right)V_1 - \left(\frac{1}{30}\right)V_2 = 0$$

$$-\left(\frac{1}{30}\right)V_1 + \left(\frac{1}{30}+\frac{1}{300}+\frac{1}{40}\right)V_2 - \left(\frac{1}{40}\right)V_3 = 0$$

$$-\left(\frac{1}{40}\right)V_2 + \left(\frac{1}{40}+\frac{1}{300}\right)V_3 = 0$$

Place in matrix form

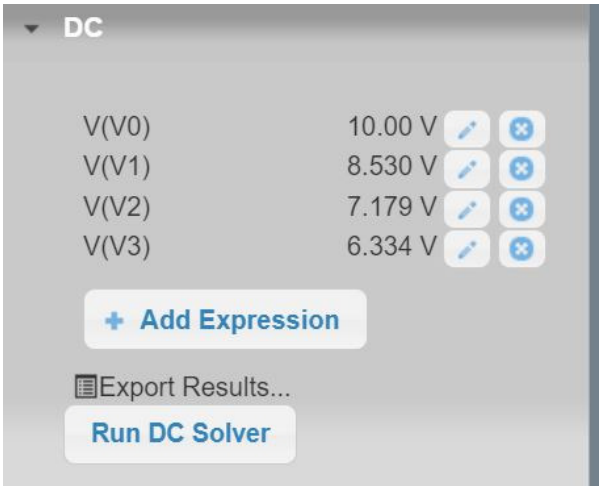
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{20}\right) & \left(\frac{1}{20}+\frac{1}{300}+\frac{1}{30}\right) & \left(\frac{-1}{30}\right) & 0 \\ 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{300}+\frac{1}{40}\right) & \left(\frac{-1}{40}\right) \\ 0 & 0 & \left(\frac{-1}{40}\right) & \left(\frac{1}{40}+\frac{1}{300}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

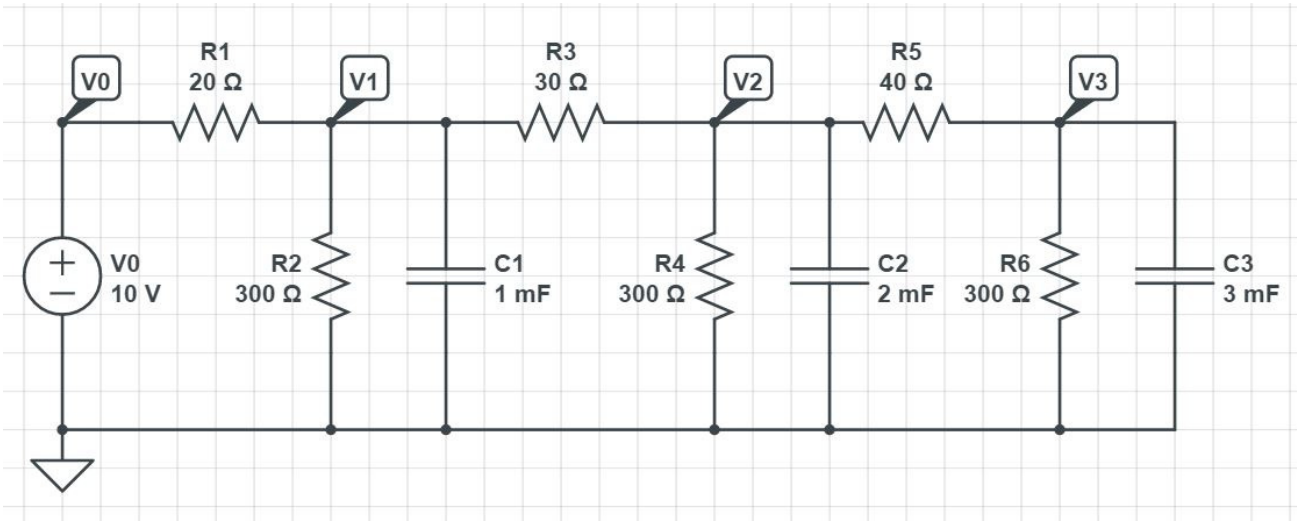
solve using matlab.

```
a1 = [1,0,0,0];
a2 = [-1/20,1/20+1/300+1/30,-1/30,0];
a3 = [0,-1/30,1/30+1/300+1/40,-1/40];
a4 = [0,0,-1/40,1/40+1/300];
A = [a1;a2;a3;a4]

    1.0000         0         0         0
   -0.0500    0.0867   -0.0333         0
         0   -0.0333    0.0617   -0.0250
         0         0   -0.0250    0.0283

B = [10;0;0;0];
V = inv(A)*B

V0   10.0000
V1    8.5304
V2    7.1791
V3    6.3345

>>
```

5) Check your results in CircuitLab

The voltages match





6) Assume V0 is a 10V, 5 rad/sec (0.796Hz)

$$V_0 = 10\sin(5t)$$

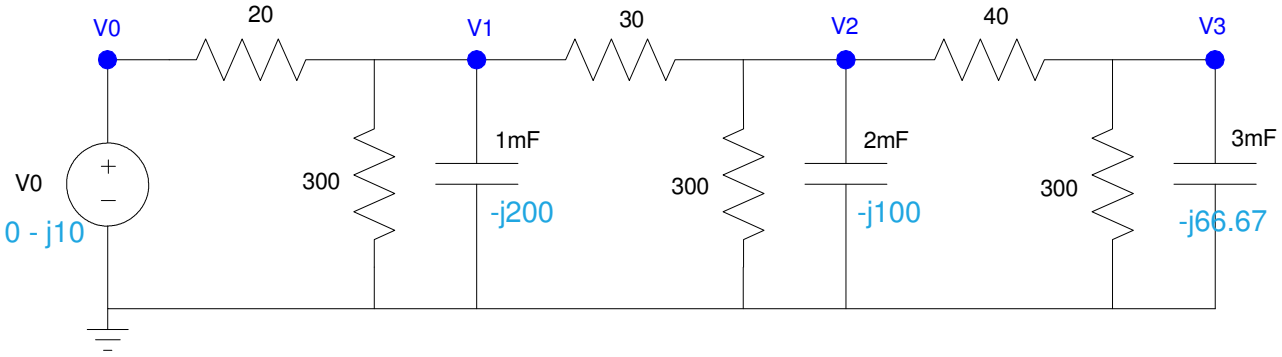
a) Determine the impedances of each element at 5 rad/sec

$$\omega = 5$$

$$V_0 \rightarrow 0 - j10$$

$$R \rightarrow R$$

$$C \rightarrow \frac{1}{j\omega C}$$



b) Write the votlage node equations

$$V_0 = -j10$$

$$\left(\frac{V_1 - V_0}{20}\right) + \left(\frac{V_1}{300}\right) + \left(\frac{V_1}{-j200}\right) + \left(\frac{V_1 - V_2}{30}\right) = 0$$

$$\left(\frac{V_2 - V_1}{30}\right) + \left(\frac{V_2}{300}\right) + \left(\frac{V_2}{-j100}\right) + \left(\frac{V_2 - V_3}{40}\right) = 0$$

$$\left(\frac{V_3 - V_2}{40}\right) + \left(\frac{V_4}{300}\right) + \left(\frac{V_3}{-j66.67}\right) = 0$$

c) Solve for V1, V2, and V3 as complex numbers

This will be the same as before but with one more term (the complex term from the capacitors)

Group terms

$$V_0 = -j10$$

$$-\left(\frac{1}{20}\right)V_0 + \left(\frac{1}{20}+\frac{1}{300}+\frac{1}{-J200}+\frac{1}{30}\right)V_1 - \left(\frac{1}{30}\right)V_2 = 0$$

$$-\left(\frac{1}{30}\right)V_1 + \left(\frac{1}{30}+\frac{1}{300}+\frac{1}{-J100}+\frac{1}{40}\right)V_2 - \left(\frac{1}{40}\right)V_3 = 0$$

$$-\left(\frac{1}{40}\right)V_2 + \left(\frac{1}{40}+\frac{1}{300}+\frac{1}{-J66.67}\right)V_3 = 0$$

place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{20}\right) & \left(\frac{1}{20}+\frac{1}{300}+\frac{1}{-j200}+\frac{1}{30}\right) & \left(\frac{-1}{30}\right) & 0 \\ 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{300}+\frac{1}{-j200}+\frac{1}{40}\right) & \left(\frac{-1}{40}\right) \\ 0 & 0 & \left(\frac{-1}{40}\right) & \left(\frac{1}{40}+\frac{1}{300}+\frac{1}{-j66.67}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
a1 = [1,0,0,0];
a2 = [-1/20,1/20+1/300+1/30+1/(-j*200),-1/30,0];
a3 = [0,-1/30,1/30+1/300+1/40+1/(-j*100),-1/40];
a4 = [0,0,-1/40,1/40+1/300+1/(-j*66.67)];
A = [a1;a2;a3;a4]
B = [-j*10;0;0;0];
V = inv(A)*B

V0         0 -10.0000i
V1   -1.5652 - 7.2895i
V2   -2.9760 - 4.1875i
V3   -3.5789 - 1.8002i
```

d) Express V1, V2, and V3 in terms of sine and cosine function:

- hint: V1 = a + jb (phasor representation) means $V_1(t) = a\cos(5t) - b\sin(5t)$

```
V0 = 10 * sin(5t)
V1 = -1.5652 cos(5t) + 7.2895 sin(5t)
V2 = -2.9760 cos(5t) + 4.1875 sin(5t)
V3 = -3.5898 cos(5t) + 1.8002 sin(5t)
```

7) Check your results in CircuitLab using a transient simulation for 6 seconds

Note: polar form is easier to see in lab (or with CircuitLab)

```
>> abs(V)              CircuitLab

   10.0000             10.000V
    7.4557              7.442V
    5.1373              5.113V
    4.0062              3.962V

>> angle(V)*180/pi

  -90.0000             this is the delay to the peak in degrees
 -102.1182
 -125.4008
 -153.2969
```