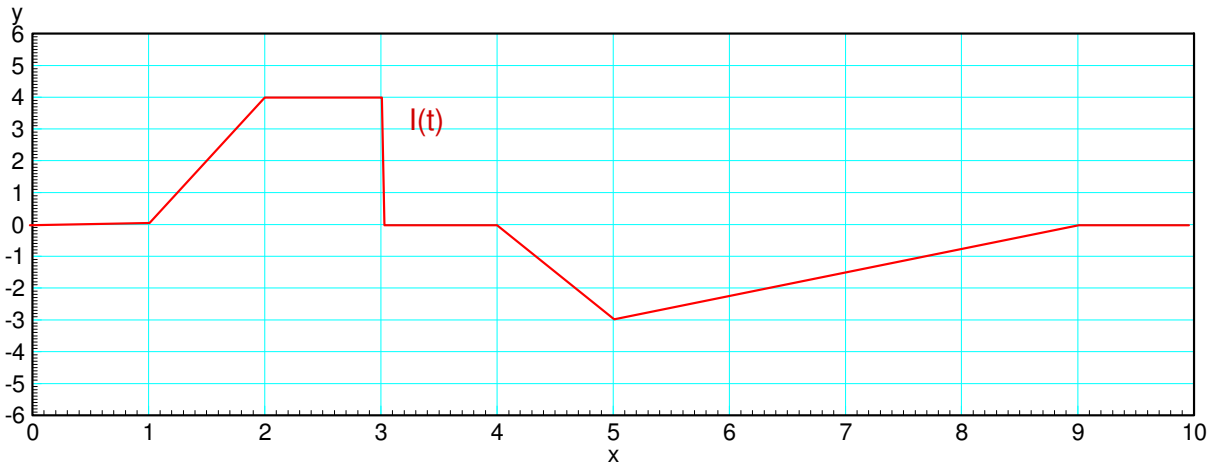
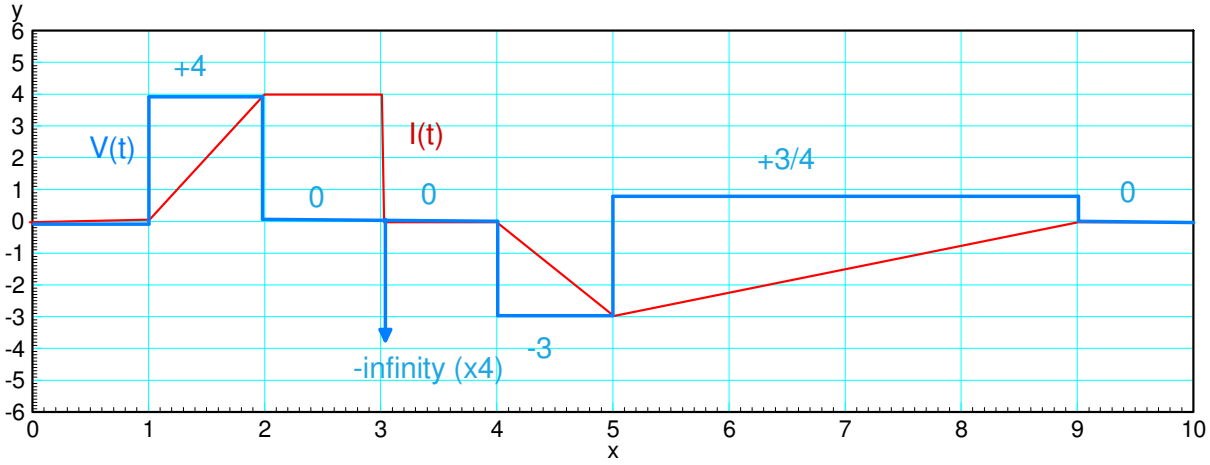
ECE 111 - Homework #8

Week #8: ECE 351 Electromagnetics - - Due 11am Tuesday, October 18th

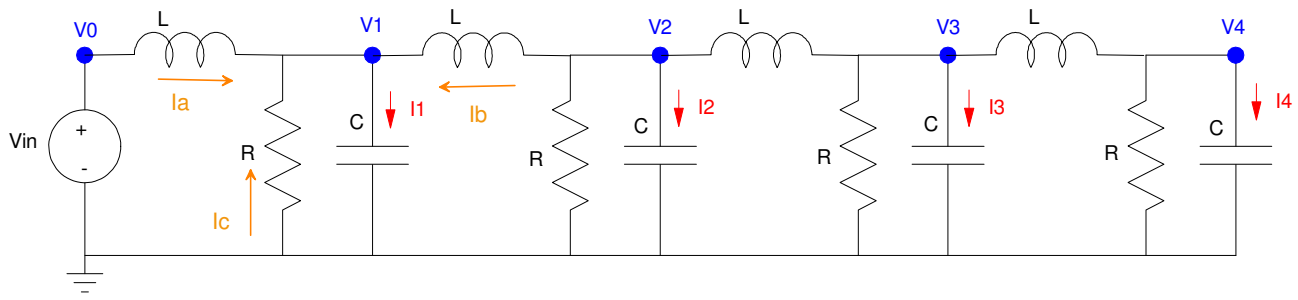
1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

$$V = L\frac{dI}{dt}$$





Problem 2-3) 4-Node RLC Circuit



R = 1000Ω, C = 0.1F, L = 0.3H. Repeat for 30 nodes for problems 4-6

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)

$$(V_0 - V_1) = L\dot{I}_a$$

$$(V_2 - V_1) = L\dot{I}_b$$

$$I_1 = C\dot{V}_1 = I_a + I_b + I_c$$

$$C\ddot{V}_1 = \dot{I}_a + \dot{I}_b + \dot{I}_c$$

$$C\ddot{V}_1 = \left(\frac{V_0 - V_1}{L}\right) + \left(\frac{V_2 - V_1}{L}\right) + \left(\frac{0 - \dot{V}_1}{R}\right)$$

Simplifying and repeating for all four nodes (note #4 misses one term since there is no node 5)

$$\ddot{V}_1 = \left(\frac{1}{LC}\right)V_0 - \left(\frac{2}{LC}\right)V_1 + \left(\frac{1}{LC}\right)V_2 - \left(\frac{1}{RC}\right)\dot{V}_1$$

$$\ddot{V}_2 = \left(\frac{1}{LC}\right)V_1 - \left(\frac{2}{LC}\right)V_2 + \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{RC}\right)\dot{V}_2$$

$$\ddot{V}_3 = \left(\frac{1}{LC}\right)V_2 - \left(\frac{2}{LC}\right)V_3 + \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)\dot{V}_3$$

$$\ddot{V}_4 = \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)\dot{V}_4$$

Plugging in numbers

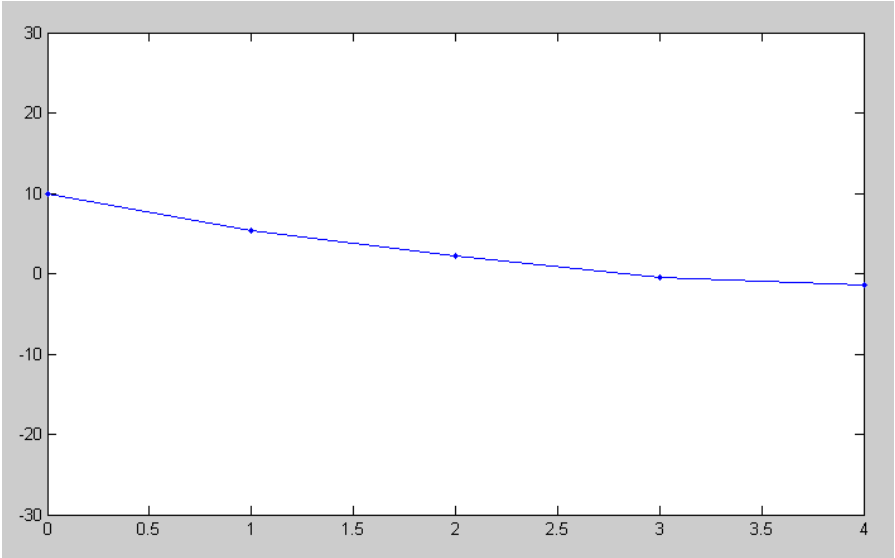
$$\ddot{V}_1 = 33.33V_0 - 66.67V_1 + 33.33V_2 - 0.01\dot{V}_1$$

$$\ddot{V}_2 = 33.33V_1 - 66.67V_2 + 33.33V_3 - 0.01\dot{V}_2$$

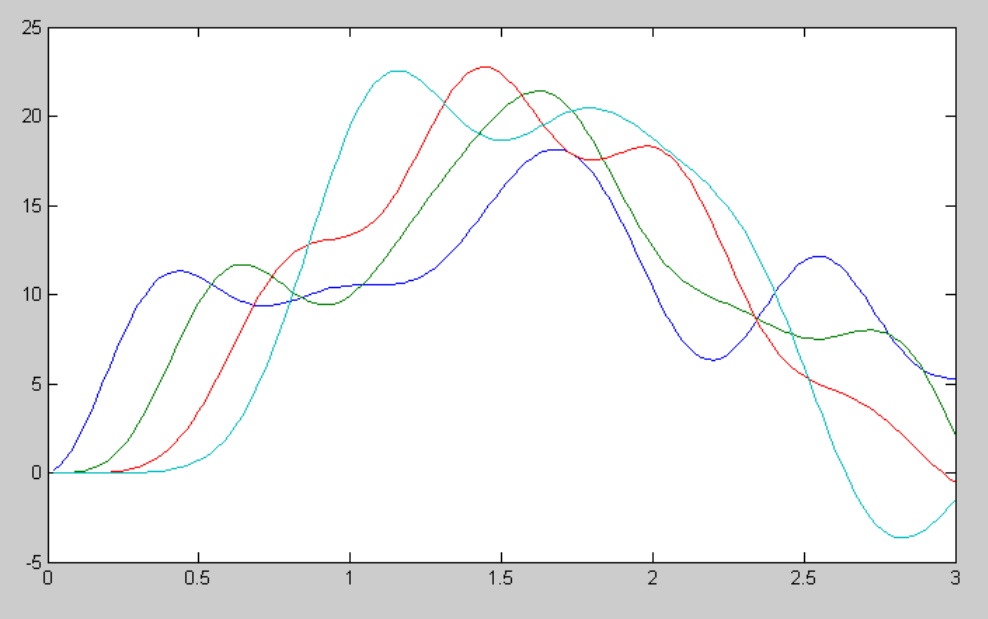
$$\ddot{V}_3 = 33.33V_2 - 66.67V_3 + 33.33V_4 - 0.01\dot{V}_3$$

$$\ddot{V}_4 = 33.33V_3 - 33.33V_4 - 0.01\dot{V}_4$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*



Node voltages at t = 3.00 seconds



Voltages vs. Time

Code:

```
% ECE 111 Lecture #8
% 4-Stage RLC Filter

V0 = 10;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;

dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;

V = [];

t = 0;
dt = 0.02;

while(t < 2.99)

   ddV1 = 33.33*V0 - 66.67*V1 + 33.33*V2 - 0.01*dV1;
   ddV2 = 33.33*V1 - 66.67*V2 + 33.33*V3 - 0.01*dV2;
   ddV3 = 33.33*V2 - 66.67*V3 + 33.33*V4 - 0.01*dV3;
   ddV4 = 33.33*V3 - 33.33*V4            - 0.01*dV4;

   dV1 = dV1 + ddV1*dt;
   dV2 = dV2 + ddV2*dt;
   dV3 = dV3 + ddV3*dt;
   dV4 = dV4 + ddV4*dt;

   V1 = V1 + dV1*dt;
   V2 = V2 + dV2*dt;
   V3 = V3 + dV3*dt;
   V4 = V4 + dV4*dt;

   t = t + dt;

   plot([0,1,2,3,4],[V0,V1,V2,V3,V4],'.-');
   ylim([-30,30]);
   clc
   disp(t)
   pause(0.01);

   V = [V; V1, V2, V3, V4];

   end

t = [1:length(V)]' * dt;
plot(t,V);
```

Problem 4-6) 30-Node RLC Circuit (hint: modify the program Wave.m)

4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 8 seconds (just after the reflection) for $1 / R_{30}C = 0.01$

This is where matricies and for-loops are useful.

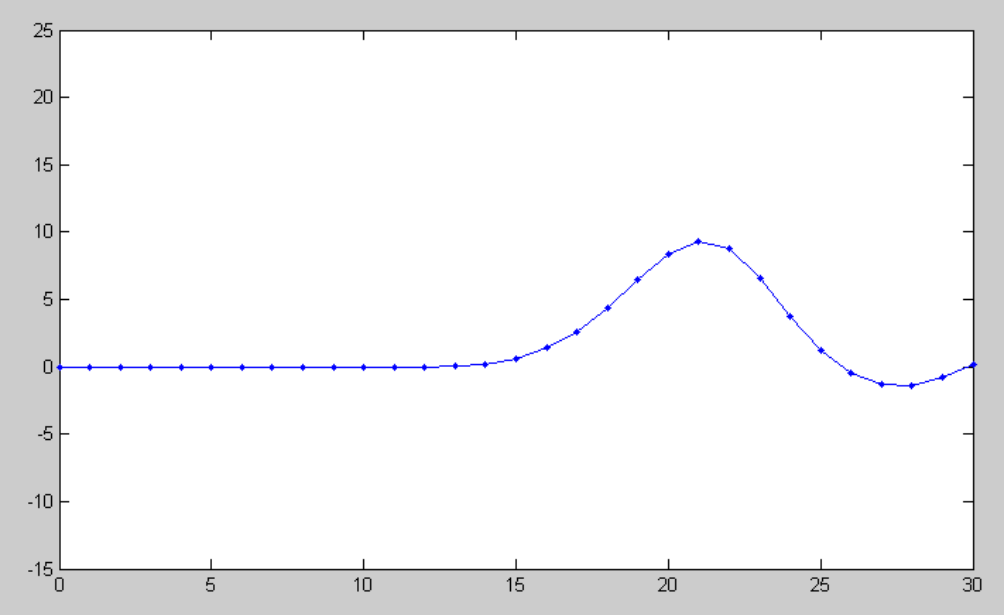
- You could take the previous code and repeat each section 30 times, or
- Place in a for-loop that counts to 30

The latter is more efficient. The trick is

- The first node is a little different: V0 is a valid Matlab variable but V(0) is not (indicies start at 1)
- The last node has a slightly different formula.

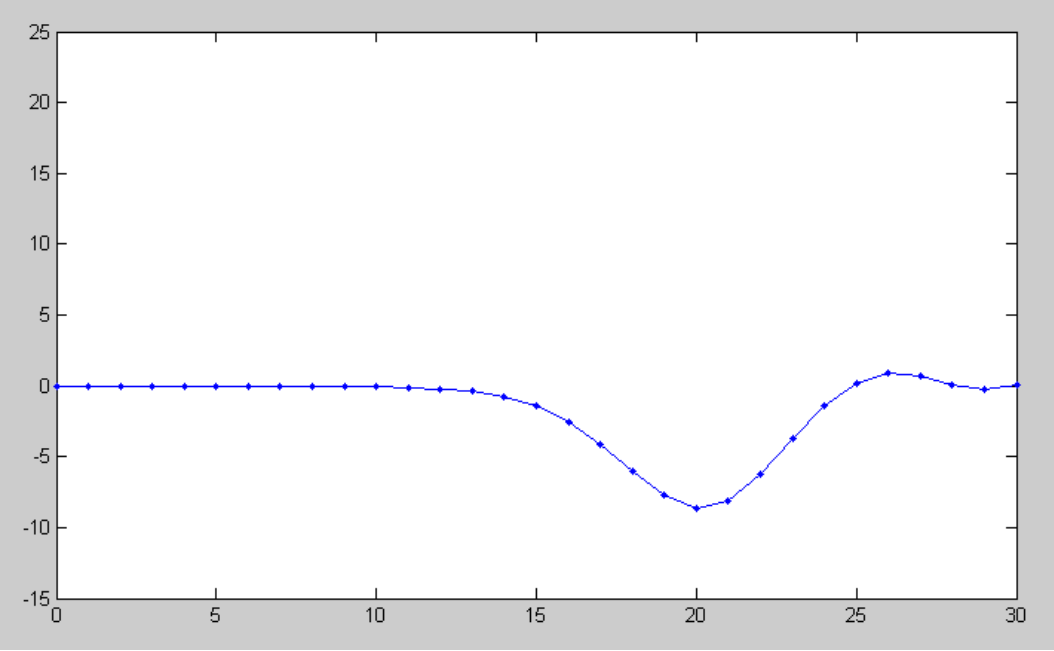
Nodes 2..29 can be placed in a for-loop however

```
% ECE 111 Lecture #8
% 30-Stage RLC Filter

N = 30;   % number of nodes

V = zeros(N,1);
dV = zeros(N,1);

t = 0;
dt = 0.02;

while(t < 8)

   if (t < 2) V0 = 10 * ( ( sin(0.5*pi*t) )^2 );
      else V0 = 0;
      end

   ddV(1) = 33.33*V0 - 66.67*V(1) + 33.33*V(2) - 0.01*dV(1);

   for i=2:N-1
      ddV(i) = 33.33*V(i-1) - 66.67*V(i) + 33.33*V(i+1) - 0.01*dV(i);
      end

   ddV(N) = 33.33*V(N-1) - 33.33*V(N) - 0.01*dV(N);
%                                  ^^^
%                               change this term


   for i=1:N
      dV(i) = dV(i) + ddV(i)*dt;
      V(i) =  V(i) +  dV(i)*dt;
      end
   t = t + dt;

   plot([0:N],[V0;V],'.-');
   ylim([-15,25]);
   clc
   disp(t)
   pause(0.01);

   end
```



Positive reflection with R30 = 1000 Ohms

5) Plot the voltage at t = 8 seconds for $1 / R_{30}C = 100$



Negative reflection when R30 = 0.1 Ohm

6) Determine experimentally R_{30} so that the reflection is almost zero

By trial-and-error

1/R30 * C = 5

R30 = 2 Ohms