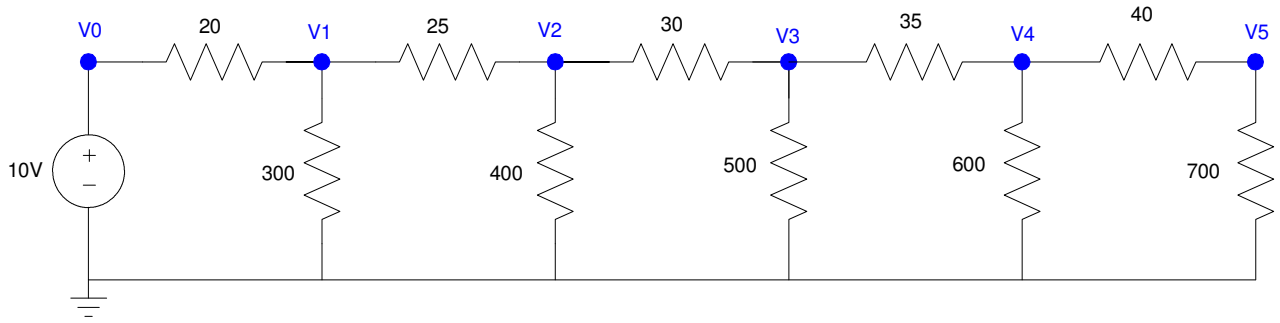
ECE 111 - Homework #6

Week #6: EE 206 Circuts I - - Due 8am Tuesday, February 22nd

1) Use Voltage Nodes write N equations for N unknowns for the following circuit.



Conservation of current: The current from any given node must sum to zero

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{20}\right) + \left(\frac{V_1}{300}\right) + \left(\frac{V_1 - V_2}{25}\right) = 0$$

$$\left(\frac{V_2 - V_1}{25}\right) + \left(\frac{V_2}{400}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{500}\right) + \left(\frac{V_3 - V_4}{35}\right) = 0$$

$$\left(\frac{V_4 - V_3}{35}\right) + \left(\frac{V_4}{600}\right) + \left(\frac{V_4 - V_5}{40}\right) = 0$$

$$\left(\frac{V_5 - V_4}{40}\right) + \left(\frac{V_5}{700}\right) = 0$$

2) Solve for the node voltages in Matlab.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{20}\right)V_0 + \left(\frac{1}{20}+\frac{1}{300}+\frac{1}{25}\right)V_1 - \left(\frac{1}{25}\right)V_2 = 0$$

$$-\left(\frac{1}{25}\right)V_1 + \left(\frac{1}{25}+\frac{1}{400}+\frac{1}{30}\right)V_2 - \left(\frac{1}{30}\right)V_3 = 0$$

$$-\left(\frac{1}{30}\right)V_2 + \left(\frac{1}{30}+\frac{1}{500}+\frac{1}{35}\right)V_3 - \left(\frac{1}{35}\right)V_4 = 0$$

$$-\left(\frac{1}{35}\right)V_3 + \left(\frac{1}{35}+\frac{1}{600}+\frac{1}{40}\right)V_4 - \left(\frac{1}{40}\right)V_5 = 0$$

$$-\left(\frac{1}{40}\right)V_4 + \left(\frac{1}{40}+\frac{1}{700}\right)V_5 = 0$$

Place in matrix form

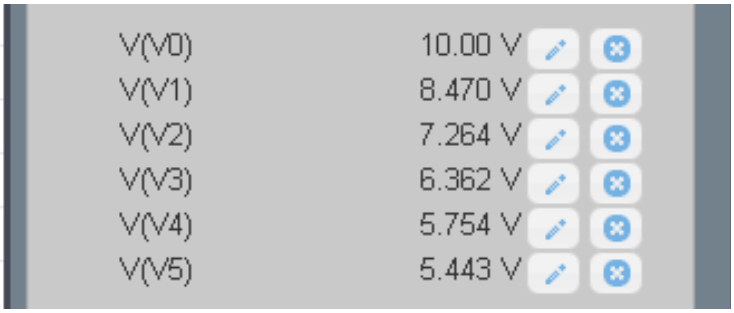
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \left(\frac{-1}{20}\right) & \left(\frac{1}{20}+\frac{1}{300}+\frac{1}{25}\right) & \left(\frac{-1}{25}\right) & 0 & 0 & 0 \\ 0 & \left(\frac{-1}{25}\right) & \left(\frac{1}{25}+\frac{1}{400}+\frac{1}{30}\right) & \left(\frac{-1}{30}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{500}+\frac{1}{35}\right) & \left(\frac{-1}{35}\right) & 0 \\ 0 & 0 & 0 & \left(\frac{-1}{35}\right) & \left(\frac{1}{35}+\frac{1}{600}+\frac{1}{40}\right) & \left(\frac{-1}{40}\right) \\ 0 & 0 & 0 & 0 & \left(\frac{-1}{40}\right) & \left(\frac{1}{40}+\frac{1}{700}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

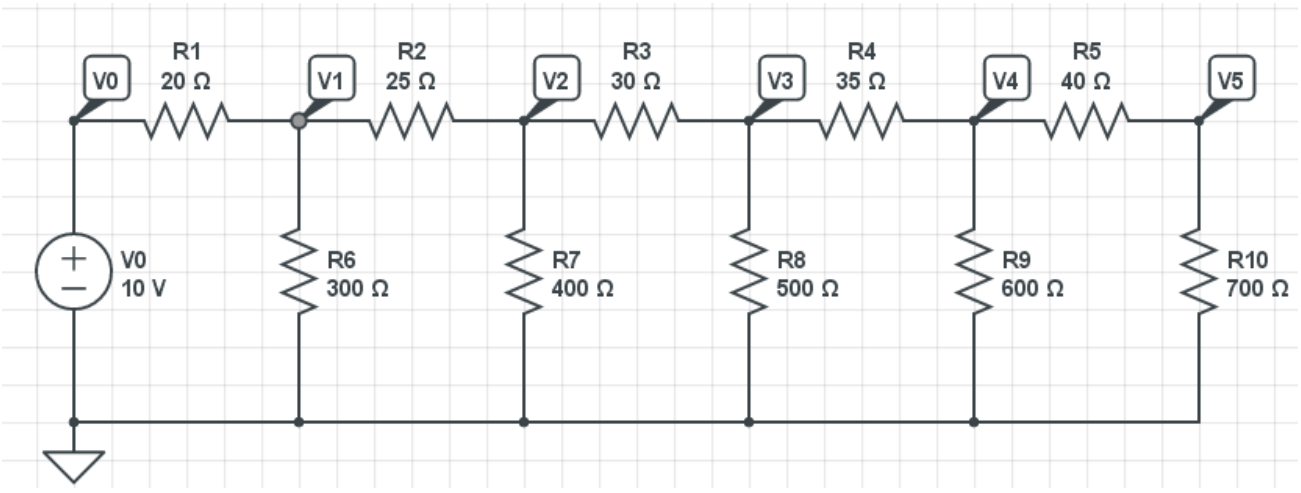
Plug into Matlab and solve

```
>> a1 = [1,0,0,0,0,0];
>> a2 = [-1/20,1/20+1/300+1/25,-1/25,0,0,0];
>> a3 = [0,-1/25,1/25+1/400+1/30,-1/30,0,0];
>> a4 = [0,0,-1/30,1/30+1/500+1/35,-1/35,0];
>> a5 = [0,0,0,-1/35,1/35+1/600+1/40,-1/40];
>> a6 = [0,0,0,0,-1/40,1/40+1/700];
>> A = [a1;a2;a3;a4;a5;a6]

A =

    1.0000         0         0         0         0         0
   -0.0500    0.0933    0.0400         0         0         0
         0   -0.0400    0.0758   -0.0333         0         0
         0         0   -0.0333    0.0639   -0.0286         0
         0         0         0   -0.0286    0.0552   -0.0250
         0         0         0         0   -0.0250    0.0264
```

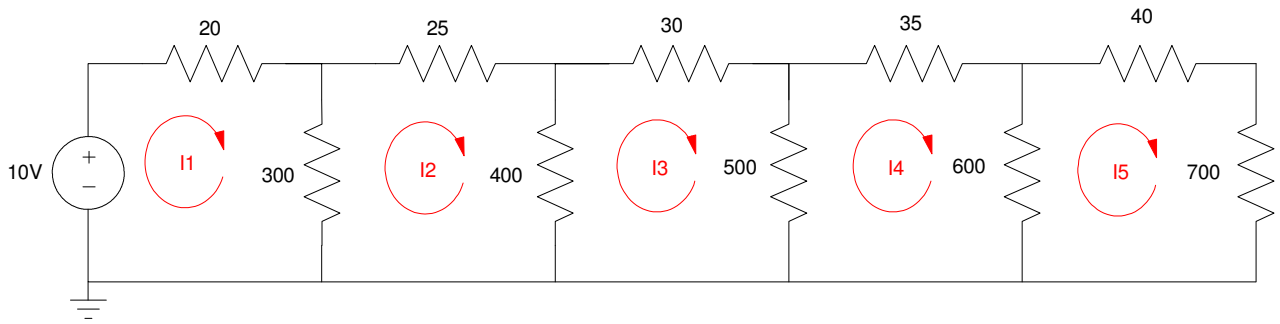
```
>> B = [10;0;0;0;0;0]

     10
      0
      0
      0
      0
      0

>> V = inv(A)*B

V0    10.0000
V1     8.4703
V2     7.2641
V3     6.3615
V4     5.7537
V5     5.4427

>>
```

3) Check your answers in CircuitLab





4) Use Current Loops to write N equations for N unknowns for the following circuit.



Conservation of Voltage: The sum of the voltages around any closed path must be zero.

$$-10 + 20I_1 + 300(I_1 - I_2) = 0$$

$$300(I_2 - I_1) + 25I_2 + 400(I_2 - I_3) = 0$$

$$400(I_3 - I_2) + 30I_3 + 500(I_3 - I_4) = 0$$

$$500(I_4 - I_3) + 35I_4 + 600(I_4 - I_5) = 0$$

$$600(I_5 - I_4) + 40I_5 + 700(I_5) = 0$$

5) Solve for the currents in Matlab

Group terms

$$320I_1 - 300I_2 = 10$$

$$-300I_1 + 725I_2 - 400I_3 = 0$$

$$-400I_2 + 930I_3 - 500I_4 = 0$$

$$-500I_3 + 1135I_4 - 600I_5 = 0$$

$$-600I_4 + 1340I_5 = 0$$

Place in matrix form

$$\begin{bmatrix} 320 & -300 & 0 & 0 & 0 \\ -300 & 725 & -400 & 0 & 0 \\ 0 & -400 & 930 & -500 & 0 \\ 0 & 0 & -500 & 1135 & -600 \\ 0 & 0 & 0 & -600 & 1340 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> A = [320,-300,0,0,0 ; -300,725,-400,0,0 ; 0,-400,930,-500,0];
>> A = [A ; 0,0,-500,1135,-600 ; 0,0,0,-600,1340]

          320        -300           0           0           0
         -300         725        -400           0           0
            0        -400         930        -500           0
            0           0        -500        1135        -600
            0           0           0        -600        1340

>> B = [10;0;0;0;0]

     10
      0
      0
      0
      0

>> I = inv(A)*B
>> mA = I*1000
```

I1 76.4827 mA
I2 48.2482 mA
I3 30.0879 mA
I4 17.3649 mA
I5 7.7753 mA

6) Check your answers in CircuitLab.

I(R1.nA)	76.48 mA
I(R2.nA)	48.25 mA
I(R3.nA)	30.09 mA
I(R4.nA)	17.36 mA
I(R5.nA)	7.775 mA