ECE 111 - Homework #1

Week #1: Algebra. Due 11am Tuesday, August 30th

functions *poly* and *roots*:

1) Use MATLAB, find the roots the the following polynomials:

a) $$x^3 - 55x^2 + 1004x - 6080 = 0$$

In Matlab, solving two different ways:

```
>> P = [1,-55,1004,-6080]

P =          1         -55        1004       -6080

>> roots(P)

   20.0000
   19.0000
   16.0000

>> roots([1,-55,1004,-6080])

   20.0000
   19.0000
   16.0000

>>
```

b) $$x^4 - 24x^3 + 209x^2 - 786x + 1080 = 0$$

```
>> P = [1,-24,209,-786,1080]

P =          1         -24          209         -786          1080

>> roots(P)

    9.0000
    6.0000
    5.0000
    4.0000
```

c) $$x^5 + 8x^4 - 49x^3 - 308x^2 + 708x + 2160 = 0$$

```
 >> P = [1,8,-49,-308,708,2160]

P =          1            8          -49         -308           708          2160

>> roots(P)

   -9.0000
   -6.0000
    5.0000
    4.0000
   -2.0000
```

2) Use Matlab to multiply our the following polynomials.

a) $(x-4)(x+3)(x-10)(x+9)=0$

```
>> R = [4,-3,10,-9]

R =      4    -3    10    -9

>> P = poly(R)

P =            1           -2         -101          102         1080

>> roots(P)

   10.0000
   -9.0000
    4.0000
   -3.0000
```

meaning

$$\boxed{x^4-2x^3-101x^2+102x+1080=0}$$

b) $(x-3)(x-4)(x-5)(x-6)(x+1)(x+4)=0$

```
>>  R = [3,4,5,6,-1,-4]

R =      3      4      5      6     -1     -4

>> P = poly(R)

P =    1         -13          33          181         -874          432          1440

>> roots(P)

   -4.0000
    6.0000
    5.0000
    4.0000
    3.0000
   -1.0000

>>
```

meaning

$$\boxed{x^6-13x^5+33x^4+181x^3-874x^2+432x+1440=0}$$

Graphing in Matlab

3) Plot the two functions in Matlab and determine all solutions in the range of -4 < x < +4

$$y = \sin(x) \cdot \cos(3x)$$

$$y = \left(\frac{x(x-2)}{10}\right)$$

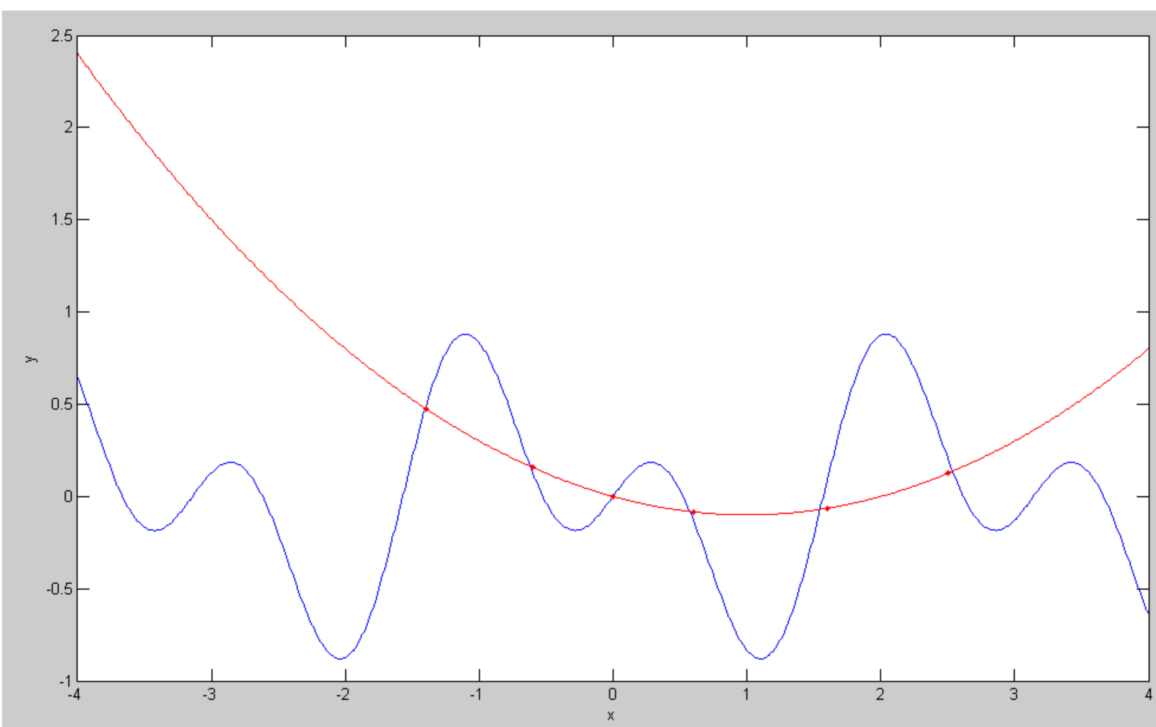
In Matlab

```
>> x = [-4:0.01:4]';
>> y1 = sin(x) .* cos(3*x);
>> y2 = x .* (x-2)/10;
>> plot(x,y1,'b',x,y2,'r');
>> xlabel('x');
>> ylabel('y');
```

From the graph, there are six solutions: x = {-1.4, -0.6, -0.1, 0.6, 1.6, 2.5,)

```
>> y3 = x .* (x-2)/10;
>> [x,y3]

      x            y
   -1.4000     0.4760
   -0.6000     0.1560
         0          0
    0.6000    -0.0840
    1.6000    -0.0640
    2.5000     0.1250
```




```
>>
```

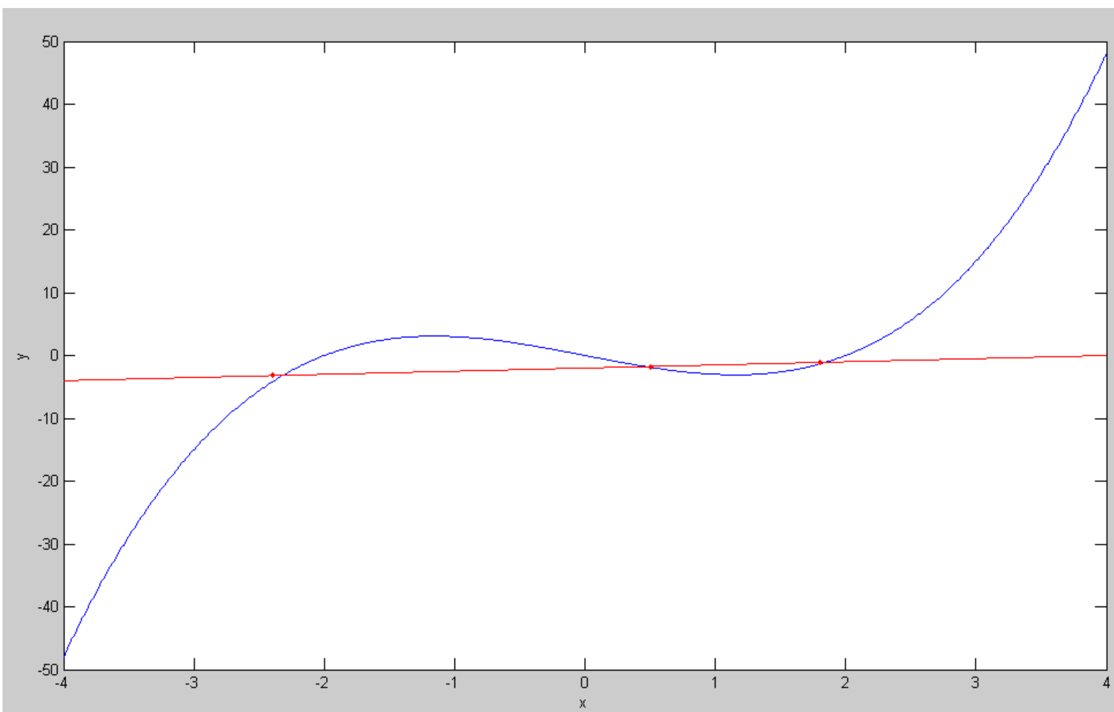
4) Plot the two functions in Matlab and determine all solutions in the range of -4 < x < +4

$$y = (x-2)(x)(x+2)$$

$$y = \frac{x}{2} - 2$$

```
>> x = [-4:0.01:4]';
>> y1 = (x-2) .* (x) .* (x+2);
>> y2 = x/2 - 2;
>> plot(x,y1,x,y2);
>> x3 = [-2.4,0.5,1.8]';
>> y3 = x3/2 - 2;
>> plot(x,y1,'b',x,y2,'r',x3,y3,'r.')
>> xlabel('x');
>> ylabel('y');
>> [x3,y3]

     x          y
  -2.4000   -3.2000
   0.5000   -1.7500
   1.8000   -1.1000
```



Monte-Carlo Simulations:

Two teams, A and B, are playing a game. Team A has a

- 50% chance of winning any given game (+1 point)
- 20% chance of a tie (+1/2 point), and
- 30% chance of a loss (+0 points)

5) For Loops: Suppose the two teams play a 9-game match. The match winner is whoever has 5 wins or more. Determine the probability that

- Team A wins the match (5 or more points),
- There is a tie (Team A has 4.5 points), and
- Team A loses (4 points or less)

matlab code:

```
Wins = 0;
Ties = 0;
Loss = 0;

for i=1:1e5
    A = 0;
    for n=1:9
        R = rand;
        if(R < 0.5) A = A + 1;
        elseif(R < 0.7) A = A + 0.5;
        else A = A + 0;
        end
    end
    if(A > 4.5) Wins = Wins + 1;
    elseif(A == 4.5) Ties = Ties + 1;
    else Loss = Loss + 1;
    end
end
disp([Wins, Ties, Loss]/1e5)
```

Results

wins	ties	losses
0.6923	0.1158	0.1920
0.6957	0.1139	0.1903
0.6954	0.1135	0.1911
0.6929	0.1139	0.1933
0.6947	0.1132	0.1921

It's about

- A 69% chance that A wins
- 11% chance of a tie
- 19% chance B wins

6) While Loops: Suppose the two teams play until one team is up by 2 points. Determine the probability that team A will win the match.

hint: use a while-loop and keep looping until one team is up by 2 games.

Code:

```
Wins = 0;
Loss = 0;

for i=1:1e5
    A = 0;
    while(abs(A) < 2)
        R = rand;
        if(R < 0.5) A = A + 1;
        elseif(R < 0.7) A = A + 0;
        else A = A - 1;
        end
    end
    if(A > 0) Wins = Wins + 1;
    else Loss = Loss + 1;
    end
end
disp([Wins, Loss]/1e5)
```

Results

Wins	Losses
0.7352	0.2648
0.7345	0.2655
0.7349	0.2651
0.7391	0.2609
0.7343	0.2657
0.7339	0.2661
0.7369	0.2631
0.7330	0.2670
0.7341	0.2659

There is about a

- 73% chance that A wins
- 26% chance that B wins
- 0% chance of a tie

7) Gauss' Dilema: Play the following game 1000 times. (i.e. use Matlab and a for loop along with a while loop)

- It costs $20 to play. The pot starts at $1.
- Flip a coin. If you get a heads, the pot doubles. If you get a tails, the game is over and you collect the money in the pot.
- Keep flipping until you get a tails.

How much money do you expect to win (or lose) each time you play this game?

Code:

```
Winnings = 0;

for i=1:1e3
    Pot = 1;
    while(rand < 0.5)
        Pot = Pot * 2;
        end
    Winnings = Winnings + Pot - 20;
end

disp(Winnings)
```

Results:

```
      -1635
       1195
     -12492
      -6053
     -14601
     -13066
     -15663
     -14667
     -14030
     -12704
     -13415
     -15238
     -14886
      -4447
     -12455
```

On average, I'm losing about $13,000 every 1000 times I play this game

- But, the mathematics say I should win an infinite amount each time I play (on average)
- Hence the name *Gauss' Dilema*

Dice:

8a) Determine the probability distribution for the following:

- Roll five 6-sided dice and five 8-sided dice.
- The total is the sum of all of the dice.

  Y = 5d6 + 5d8

8b) What is the probability of the total being 50?

8c) What is the probability of the total being 50 or more?

Code:

```
B = 0;
C = 0;

for i = 1:1e5
    d6 = ceil(6*rand(1,5));
    d8 = ceil(8*rand(1,5));
    Y = sum(d6) + sum(d8);
    if(Y == 50) B = B + 1; end
    if(Y >= 50) C = C + 1; end
end
disp([B,C]/1e5)
```

Results

```
      = 50    >= 50
     0.0187    0.0684
     0.0198    0.0706
     0.0190    0.0697
     0.0186    0.0691
     0.0186    0.0685
     0.0192    0.0687
     0.0193    0.0694
```

There is about a

- 1.9% chance the sum will be 50
- 6.9% chance the sum will be 50 or more

9) Two people are playing a dice game:

- Player A rolls five 6-sided and five 8-sided dice and takes the total (5d6 + 5d8)
- Player B rolls two 100-sided dice and takes the lower of the two numbers.
- Whoever has the highest score wins.

Determine the probability that

- A wins
- There is a tie, and
- B wins

Code:

```
Wins = 0;
Ties = 0;
Loss = 0;

for i = 1:1e5
    d6 = ceil(6*rand(1,5));
    d8 = ceil(8*rand(1,5));
    d100 = ceil(100*rand(1,2));

    A = sum(d6) + sum(d8);
    B = min(d100);

    if(A > B) Wins = Wins + 1; end
    if(A == B) Ties = Ties + 1; end
    if(A < B) Loss = Loss + 1; end
end
disp([Wins, Ties, Loss]/1e5)
```

Results:

A wins	Tie	B wins
0.6220	0.0123	0.3657
0.6192	0.0129	0.3679
0.6246	0.0120	0.3634
0.6204	0.0122	0.3674
0.6243	0.0123	0.3635
0.6218	0.0126	0.3656
0.6231	0.0123	0.3646

There is about a

- 62% chance that A will win,
- 1.2% chance of a tie, and
- 36% chance B wins