
Biomedical Engineering

ECE 111: Intro to ECE

Jake Glower

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

Biomedical Engineering

Objective

- Model the cardiovascular system as an RLC circuit
- From the model, predict what happens if different parameters change

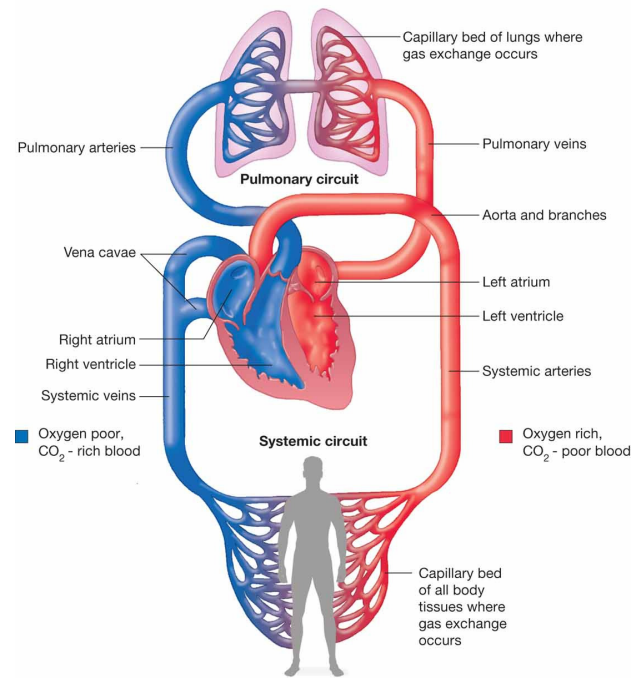
Matlab Functions

- Matlab Scripts
 - Nonlinear simulations
 - clf
 - subplot
-

Electrical Model of the cardiovascular System

The cardiovascular system consists of two circulatory systems:

- The right ventricle drives the low-pressure system (lungs), and
- The left ventricle drives the high-pressure system, (rest of body)



Modeling as an electrical circuit

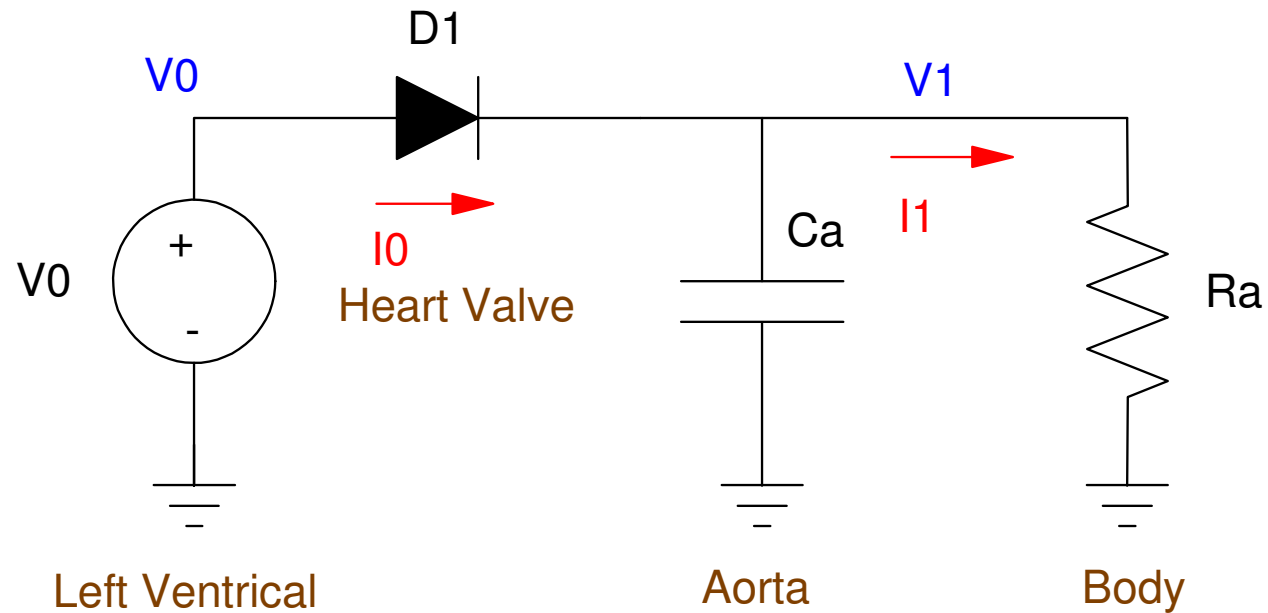
- In essence, the heart acts as a 5W source.
- Most of the work and medical problems are associated with the high-pressure side (which requires most of the energy).

If you can model the high-pressure cardiovascular system

- You have a better understanding of how the system works,
 - You can build more accurate models of the body for testing of artificial hearts
 - You can ask 'what if' questions, such as what happens if the capacitance increases, and predict the result.
 - This in turn opens new directions for drug treatment: finding drugs which affect the different parameters of the model
-

Electrical Models of the Heart

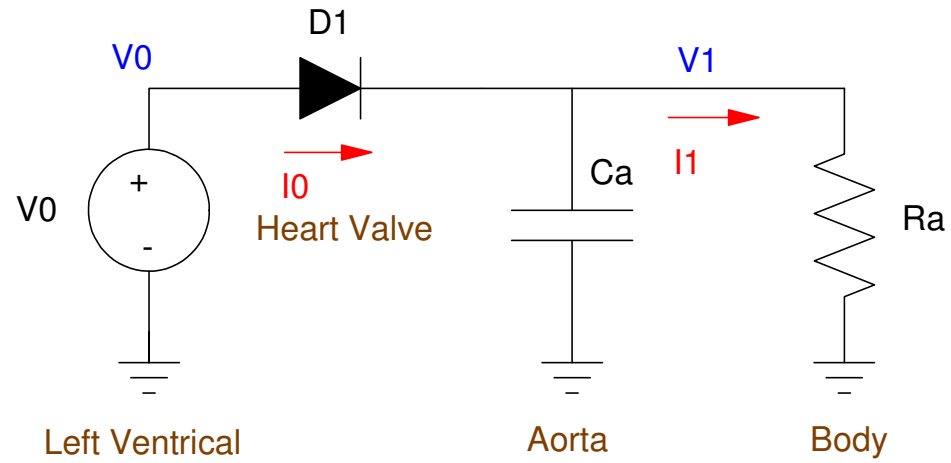
- A 2-element Winkessel model (i.e. an RC circuit), and
- A more complex 5-element model (an RLC circuit)



Heart Parameters:

Parameter	English Units	Metric Units	Circuit Dual
Pressure	89mm Hg	10.67 N/m ²	10.67V
Volume (LV)		70ml	
Volume (Aorta)		500ml	
Inertia (Aorta)			
Normal Flow		70 ml/s	70mA
Blood Inertia		0.07kg	0.07H

2-element Windkessel Model



First, determine reasonable parameters for the model.

V0: A typical blood pressure reading is 120/80.

$$V_0 = \max(0, 16 \sin(2\pi t))$$

D1: Heart Valve:

$$R_{d1} = \begin{cases} 1\Omega & V_0 > V_1 \\ 1M\Omega & V_0 < V_1 \end{cases}$$

Ra: Nominal flow is 70ml/s on average

$$R_a = \frac{13.3V}{70mA} = 190\Omega$$

C: Pressure range of 120/80 mmHg (26V .. 10.6V) is

$$C_a = 4.54mF$$

This results in the following dynamic equations for the heart:

$$I_c = C_a \frac{dV_1}{dt} = I_{d1} - \frac{V_1}{R_a}$$

Matlab Code:

```
for j=1:npt

    V0 = max(0, 16*sin(6.28*t));

    if (V0 > V1)    Rd1 = 1;
                   else Rd1 = 1e5;
                   end

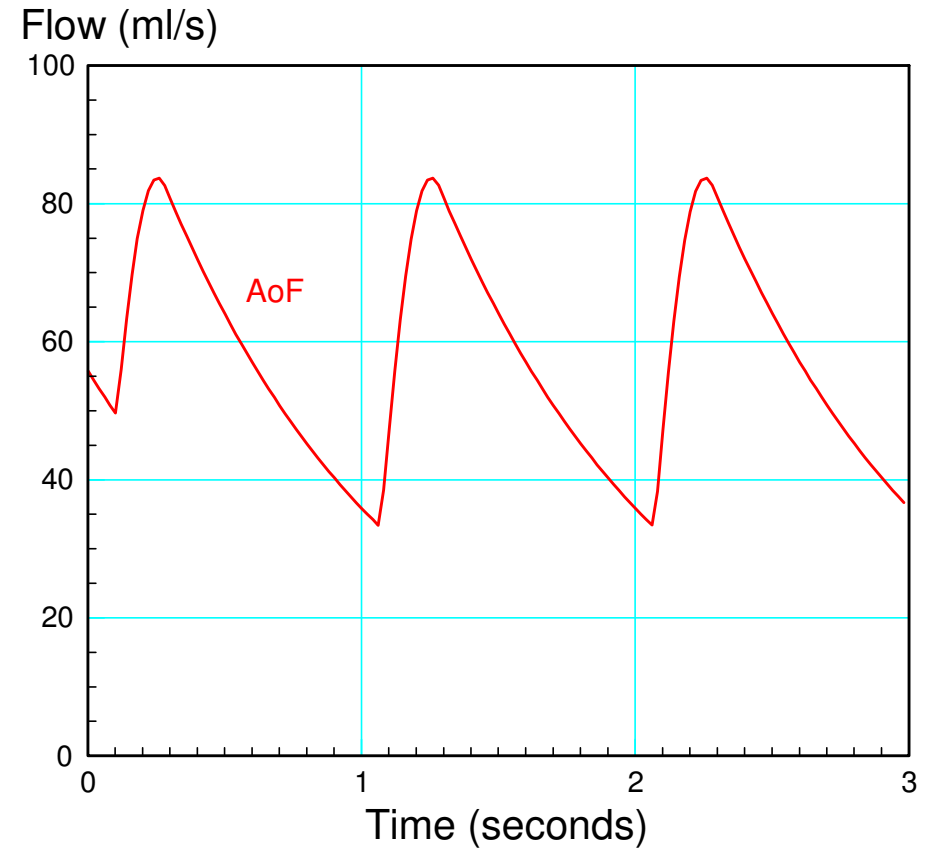
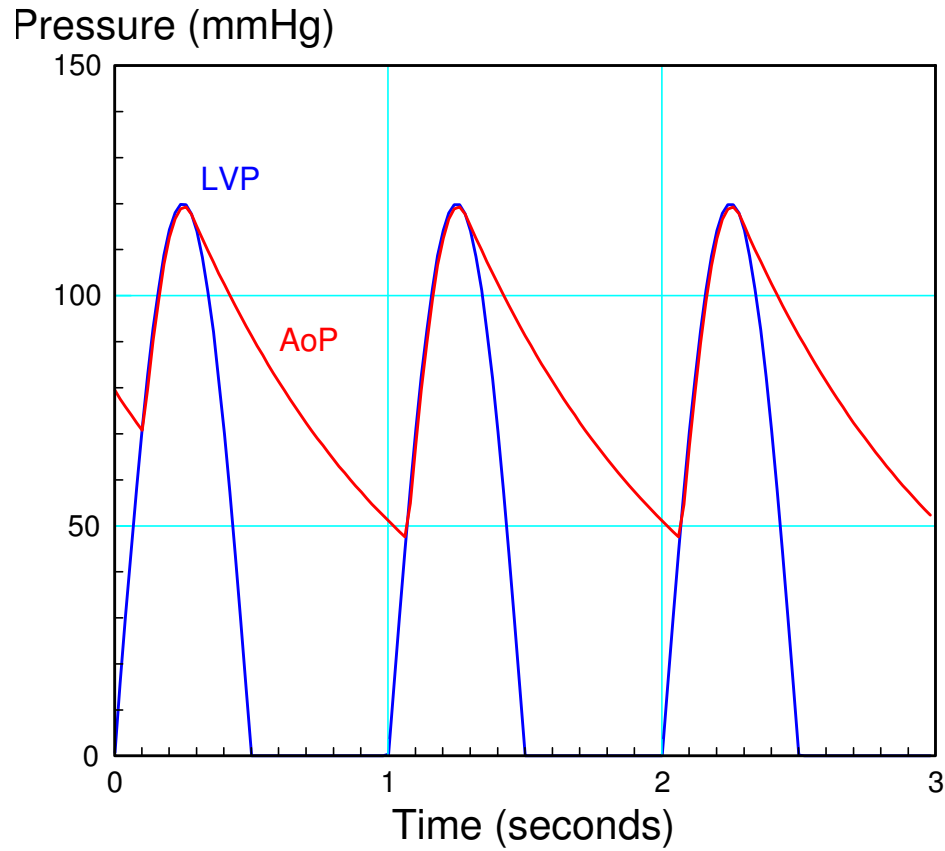
    I0 = (V0 - V1)/Rd1;
    I1 = V1 / Ra;

    dV1 = (I0 - I1)/Ca;

    t = t + dt;
    V1 = V1 + dV1*dt;

end
```

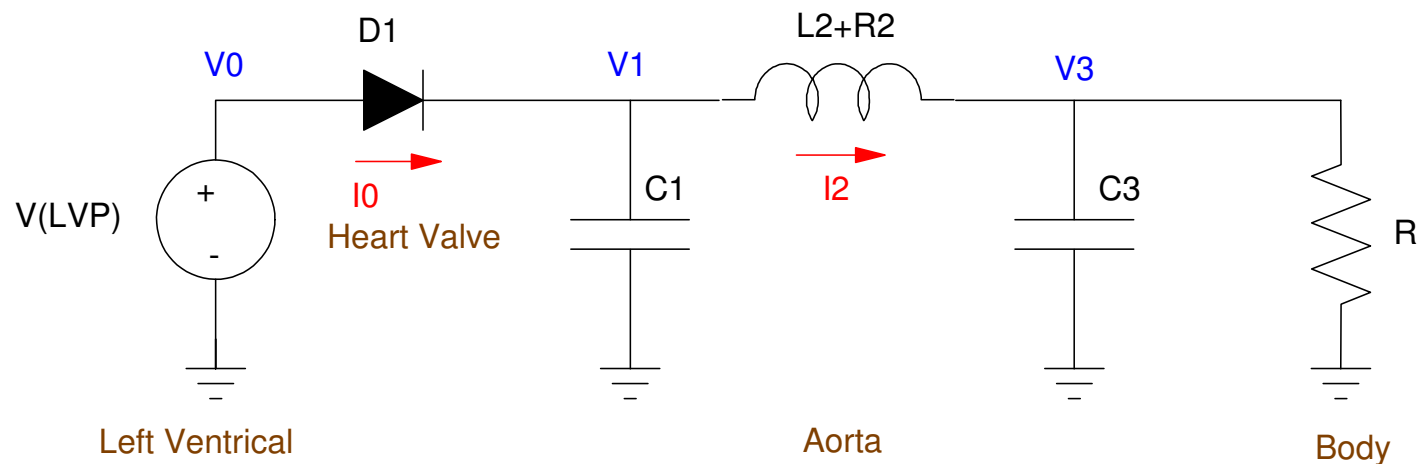
Results for Three Beats:



Improved Winkessel Model

A slightly more accurate model splits the Aorta into two sections:

- C1 accepts blood from the heart and charges up
- L2 is the inertia of the blood in this section of the aorta,
- C3 is the rest of the arterial system which pushes blood throughout the body, and
- R models the capillaries, drawing 70ml/s (70mA) on average



Computations for the parameters are as follows:

$$70mA = C_3 \frac{40mmHg}{0.8s} = C_3 \frac{5.33V}{0.8s} \quad \Rightarrow \quad C_3 = 10.5mF$$

$$C_1 = \left(\frac{70ml}{500ml} \right) \cdot 10.5mF \quad \Rightarrow \quad C_1 = 1.5mF$$

L2: $70ml = 0.07kg = 0.07H + 6 \text{ Ohms}$.

R: $R = 190\Omega$ (same as before)

V0: Model the left ventricle

$$V_0 = \max(0, 17.6 \sin(2\pi t))$$

D1: Model the valve as a variable resistor

$$R_{d1} = \begin{cases} 1\Omega & V_0 > V_1 \\ 1M\Omega & V_0 < V_1 \end{cases}$$

Dynamics equations:

$$I_0 = \frac{V_0 - V_1}{R_{d1}}$$

$$C_1 \frac{dV_1}{dt} = I_0 - I_2$$

$$L_2 \frac{dI_2}{dt} = V_1 - I_2 R_2 - V_3$$

$$C_3 \frac{dV_3}{dt} = I_2 - \frac{V_3}{R}$$

Matlab Code (main loop)

```
for j=1:npt

    V0 = max(0, 17.6*sin(6.28*t));

    if (V0 > V1)    Rd1 = 1;
                   else Rd1 = 1e5;
                   end

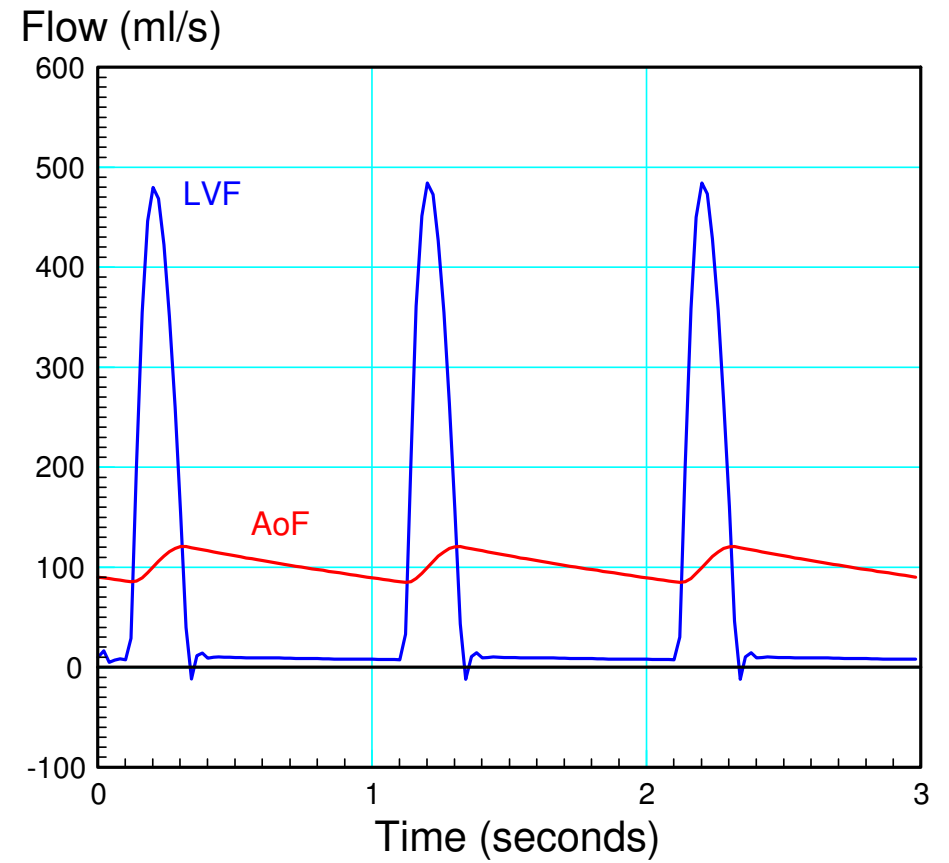
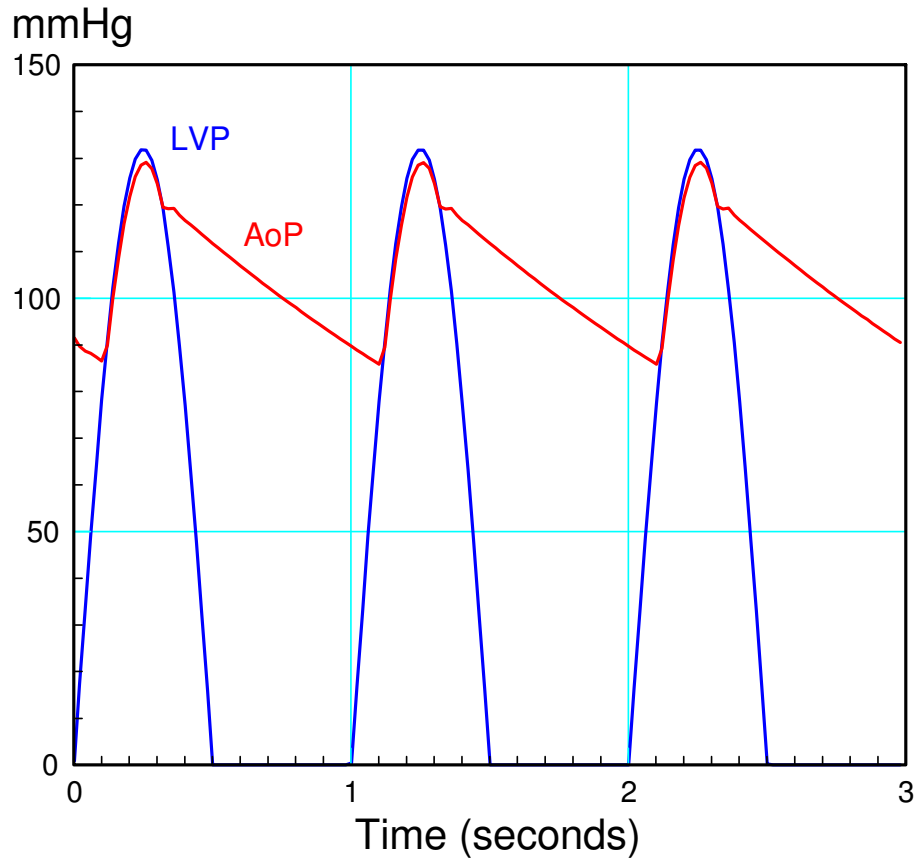
    I0 = (V0 - V1)/Rd1;

    dV1 = (I0 - I2)/C1;
    dI2 = (V1 - R2*I2 - V3)/L2;
    dV3 = (I2 - V3/R)/C3;

    t = t + dt;
    V1 = V1 + dV1*dt;
    I2 = I2 + dI2*dt;
    V3 = V3 + dV3*dt;

end
```

Result for 3 beats



What this model shows you is:

- The aortic pressure (top graph red) is smoother. This agrees with what you see better.
 - At the left ventricle (right blue), there is a dip in pressure when the valve closes. This is caused by the pressure wave reflecting back in time to help the valve close (something you also see in a healthy person)
 - The arterial flow (right red) also has a negative dip when the valve closes. This is the inertia of the blood providing suction, helping the heart to eject more blood.
-

With this model you can start to answer questions, such as

- What is the effect of Angina (the aorta expands)
- What is the effect of hardening of the arteries?
- What is the effect of obstructed flow?
- How does the body respond to the above to maintain blood flow?

