ECE 111: Handout #15

ECE 343 Random Processes: Chi-Squared & t-Test

1) A 6-sided die is rolled 90 times. Use a chi-squared test to determine probability that the die is not fair

a) Using 6 bins (each number)

Number	probability (p)	Expected Frequency (np)	Actual Frequency (N)	$\frac{(np-N)^2}{np}$
1	1/6	15	12	
2	1/6	15	10	
3	1/6	15	13	
4	1/6	15	13	
5	1/6	15	25	
6	1/6	15	17	
			Sum	

b) Using 3 bins

Number	probability (p)	Expected Frequency (np)	Actual Frequency (N)	$\frac{(np-N)^2}{np}$
1,2,3	3/6	45	35	
4,5	2/6	30	28	
6	1/6	15	17	
			Sum	

Chi-Squared Table

df	99.5%	99%	97.5%	95%	90%	10%	5%	2.5%	1%	0.5%
1	7.88	6.64	5.02	3.84	2.71	0.02	0	0	0	0
2	10.6	9.21	7.38	5.99	4.61	0.21	0.1	0.05	0.02	0.01
3	12.84	11.35	9.35	7.82	6.25	0.58	0.35	0.22	0.12	0.07
4	14.86	13.28	11.14	9.49	7.78	1.06	0.71	0.48	0.3	0.21
5	16.75	15.09	12.83	11.07	9.24	1.61	1.15	0.83	0.55	0.41
6	18.55	16.81	14.45	12.59	10.65	2.2	1.64	1.24	0.87	0.68

2) The gain of 5 transistors were measured

- { 915, 602, 963, 839, 815 }
- mean = 826.8
- standard deviation = 138.9

What is the probability that any given transistor has a gain more than 600?

What is the 98% confidence interval for the gain of any given transistor?

Student t-Table (http://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf)										
p	0.75	0.8	0.85	0.9	0.95	0.975	0.99	0.995	0.999	0.9995
1	1	1.38	1.96	3.08	6.31	12.71	31.82	63.66	318.31	636.62
2	0.82	1.06	1.39	1.89	2.92	4.3	6.97	9.93	22.33	31.6
3	0.77	0.98	1.25	1.64	2.35	3.18	4.54	5.84	10.22	12.92
4	0.74	0.94	1.19	1.53	2.13	2.78	3.75	4.6	7.17	8.61
5	0.73	0.92	1.16	1.48	2.02	2.57	3.37	4.03	5.89	6.87
infinity	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.29

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Random Processes: Chi-Squared & t-Test

1) A 6-sided die is rolled 90 times. Use a chi-squared test to determine probability that the die is not fair

a) Using 6 bins (each number)

Number	probability (p)	Expected Frequency (np)	Actual Frequency (N)	$\frac{(np-N)^2}{np}$
1	1/6	15	12	**0.6**
2	1/6	15	10	**1.67**
3	1/6	15	13	**0.27**
4	1/6	15	13	**0.27**
5	1/6	15	25	**6.67**
6	1/6	15	17	**0.27**
			Sum	**9.73**

There is about a 90% chance that the die is not fair (reject the null hypothesis)

Chi-Squared Table

df	99.5%	99%	97.5%	95%	90%	10%	5%	2.5%	1%	0.5%
5	16.75	15.09	12.83	11.07	**9.24**	1.61	1.15	0.83	0.55	0.41

b) Using 3 bins

Number	probability (p)	Expected Frequency (np)	Actual Frequency (N)	$\frac{(np-N)^2}{np}$
1,2,3	3/6	45	35	**2.22**
4,5	2/6	30	28	**0.13**
6	1/6	15	17	**0.27**
			Sum	**2.62**

There is less than a 90% change that the die is not fair (reject the null hypothesis)

Chi-Squared Table

df	99.5%	99%	97.5%	95%	90%	10%	5%	2.5%	1%	0.5%
2	10.6	9.21	7.38	5.99	**4.61**	**0.21**	0.1	0.05	0.02	0.01

2) The gain of 5 transistors were measured

- { 915, 602, 963, 839, 815 }
- mean = 826.8
- standard deviation = 138.9

What is the probability that any given transistor has a gain more than 600?

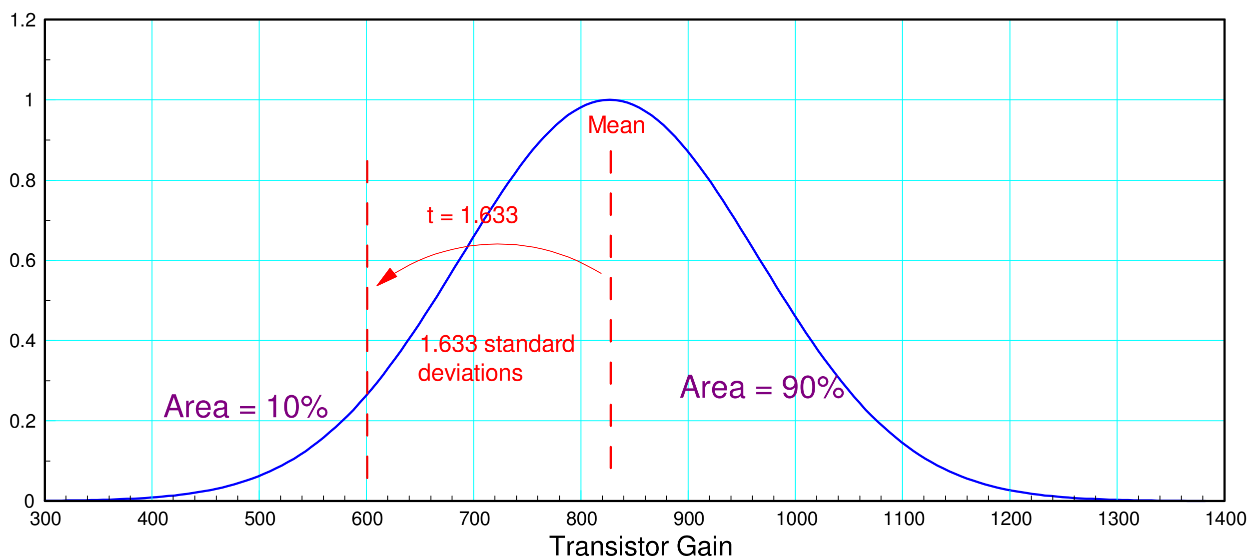
Determine the t-score

$$t = \left(\frac{\bar{x} - 600}{s}\right) = \left(\frac{826.8 - 600}{138.9}\right) = 1.633$$

From a t-table with 4 degrees of freedom, this corresponds to a probability of about 90%

There is a 90% chance that any given transistor has a gain of 600 or more

Student t-Table (http://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf)										
p	0.75	0.8	0.85	0.9	0.95	0.975	0.99	0.995	0.999	0.9995
4	0.74	0.94	1.19	**1.53**	2.13	2.78	3.75	4.6	7.17	8.61



pdf of transistor gain. 90% of the area is to the right of 600

What is the 98% confidence interval for the gain of any given transistor?

Each tail needs to be 1% (t-score is 3.75)

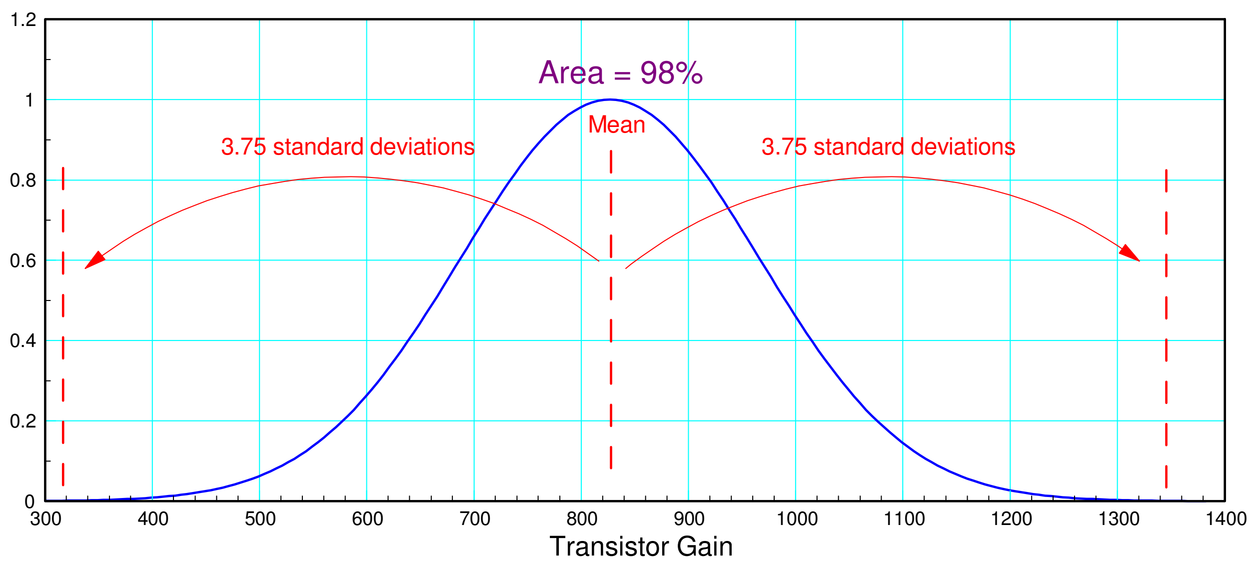
The 98% confidence interval is then

$$\bar{x} - 3.75s < gain < \bar{x} + 3.75s$$

$$826.8 - 3.75 \cdot 138.9 < gain < 826.8 + 3.75 \cdot 138.9$$

$$305.9 < gain < 1347$$

Student t-Table (http://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf)										
p	0.75	0.8	0.85	0.9	0.95	0.975	0.99	0.995	0.999	0.9995
4	0.74	0.94	1.19	1.53	2.13	2.78	**3.75**	4.6	7.17	8.61



98% Confidence Interval: 98% of transisors should have a gain in the range of (305.9, 1348)