ECE 111: Handout #14

Week #14: ECE 343 Signals and Systems

Problem 1: Determine the transfer function from the differential equation

$$y'' + 5y' + 8y = 2x' + 10x$$

Problem 2: Determine the differential equation which relates X and Y

$$Y = \left(\frac{10s+20}{s^2+6s+5}\right)X$$

Problem 3: Find y(t)

$$Y = \left(\frac{10}{(s+1)(s+3)}\right)X$$

$$x(t) = 4\cos(5t) + 2\sin(5t)$$

Solutions

Problem 1: Determine the transfer function from the differential equation

$$y'' + 5y' + 8y = 2x' + 10x$$

Assume all functions are in the form of

$$y = e^{st}$$

Then

$$\frac{dy}{dt} = s \cdot e^{st} = sy$$

sY mean *the derivative of y*

$$s^2Y + 5sY + 8Y = 2sX + 10X$$

$$(s^2 + 5s + 8)Y = (2s + 10)X$$

$$Y = \left(\frac{2s+10}{s^2+5s+8}\right)X$$

Problem 2: Determine the differential equation which relates X and Y

$$Y = \left(\frac{10s+20}{s^2+6s+5}\right)X$$

Cross multiply

$$(s^2 + 6s + 5)Y = (10s + 20)X$$

sY mean *the derivative of y*

$$y'' + 6y' + 5y = 10x' + 20x$$

Problem 3: Find y(t)

$$Y = \left(\frac{10}{(s+1)(s+3)}\right) X$$

$$x(t) = 4\cos(5t) + 2\sin(5t)$$

Solution: Express as phasors

$$s = j5$$

$$X = 4 - j2$$

$$Y = \left(\frac{10}{(s+1)(s+3)}\right)_{s=j5} (4 - j2)$$

$$Y = (-0.249 - j0.226)(4 - j2)$$

$$Y = -1.448 - j0.407$$

which is phasor shorthand for y(t)

$$y(t) = -1.488\cos(5t) + 0.407\sin(5t)$$